Simulation of fully coupled finite element analysis of nonlinear hydraulic properties in land subsidence due to groundwater pumping

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Abstract A fully coupled mathematical model of land subsidence caused by groundwater pumping was established based on the mechanics of porous seepage and the theory of fluid–solid interaction. The mathematical model employing the Galerkin finite element method was proposed to simulate the deformation dependencies of hydraulic properties due to the water pressure decrease in aquifers. This model has been verified by comparing with the known analytical solutions in the confined aquifer. Results show that the simulated drawdown and displacements match well with those of analytical solutions. To evaluate the nonlinear effects of hydraulic properties in the seepage and consolidation coupling phenomena, the numerical model is applied to an ideal three layers numerical experiment. The results show that the subsidence rate is faster than conventional groundwater theory when the nonlinear hydraulic properties (NHP) are considered in the coupled model. The reason is that the water level decline due to groundwater withdrawal induces the soil

compression and the porosity and permeability decrease. Decrease of permeability in regions adjacent to the pumping well produces hydraulic gradient and seepage forces that may result in accelerated subsidence. Therefore, the effect of the NHP is an interaction process of pore water pressure and soil consolidation deformation and should not be ignored. Further studies of various hydrogeology problems are recommended to consider the heterogeneity concerning different stratigraphic condition in the field.

Keywords Fluid–solid interaction · Land subsidence · Groundwater pumping · Nonlinear · Numerical model

Introduction

Land subsidence due to groundwater pumping has induced various problems including unequal settlement, wall cracking, pipeline damage and ground fissures in many cities around the world (Khan et al. 2013; Abidin et al. 2013). Groundwater pumping induces the fully coupled hydraulic–mechanical interaction between water flow and solid matrix in nature in many cases. As groundwater pumping take place, pressure decline in aquifers is accompanied by a change in effective stress in the solid matrix, and then compaction of the solid skeleton of the aquifer ensues. But on the other hand, when porous media deformation occurs, its physical parameters, such as porosity, permeability, will also be changed. Therefore, it is necessary to consider the coupling of water flow fluctuations and the deformation characteristics of fluid field media. This fully coupled phenomenon can be better explained through the Biot's consolidation theory which provides a more rigorous and realistic

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mathematical method to analyze the fully coupled groundwater flow and solid matrix deformation. Biot's (1941) consolidation theory has been widely applied in hydrogeology, geomechanics, engineering and many other fields. On the basis of Biot's work, a number of scholars use analytical solutions and numerical solutions to evaluate the fully coupled water flow and solid matrix deformation due to groundwater pumping. Bear and Corapcioglu (1981) proposed an analytical solution for regional land subsidence due to groundwater pumping. Safai and Pinder (1979) constructed a Galerkin finite element model to simulate the vertical and horizontal land deformation in a desaturating porous medium. Schrefler and Zhan (1993) developed a fully coupled model to simulate the slow transient phenomena involving flow of water and air in deforming porous media. Bai and Elsworth (1994) used a dual-porosity poroelastic model to represent the coupled flow and deformation behavior in fractured rock specimens. Yeh et al. (1996) developed a general Galerkin finite element model to study the behavior of land displacements due to pressure decline in aquifers. Gatmiri and Delage (1997) developed a formulation of fully coupled thermal–hydraulic–mechanical behavior of saturated porous media. Lewis and Schrefler (1978, 1998) presented a coupled model to simulate the subsidence and the pressure decline in the considered area. Kim and Parizek (1999) and Kim (2000) presented a fully coupled numerical model for groundwater flow and soil deformation due to groundwater pumping. Gambolati et al. (2000) analyzed the comparison of results obtained with the finite element of the coupled and uncoupled models, which indicated that pore pressure was rather insensitive to the coupling model within the pumped formation. Zimmerman et al. (1986) pointed out that the coupling effect was usually lies between 0.1 and 1.0 and concluded that the pore-elastic coupling effect was usually strong and could not be ignored. These numerical analyses assumed linear coupling between water pressure and solid matrix. However, the hydraulic properties of the porous media depended on the deformation, which induced nonlinear coupling. There have been several experimental and mathematical studies on the permeability and pore–volume changes with pore water pressure and soil stress, such as Zimmerman (2000) believed an increase in temperature may lead to a large increase in fluid pressure, which will then give rise to mechanical stresses and strains. Kim and Parizek (2005) used a fully coupled poroelastic numerical model to simulate the hydraulic and mechanical responses when hydraulic pumping and pumping shutoff, the result showed that poroelastic deformation caused by groundwater pumping and pumping shutoff from an aquifer could induce noticeable reverse hydraulic head fluctuations. Ouria et al.

(2009, 2011) developed a numerical model to investigate the effect of the changes of the soil permeability on the transient seepage, the result showed that the effect of the change of the permeability on the outlet flow was different depending on the coupled or uncoupled formulations. Budhu and Adiyaman (2010, 2013) developed a basic mechanics governing the changes in stress states from groundwater pumping and compared the predicted land subsidence with existing models and field data, the result showed that the predicted subsidence using the developed mechanics compared well with the field data. Results of these studies strongly indicated that the nonlinear character of hydraulic properties should be considered. Some scholars used the fully coupled flow in other fields, for instance, oil reservoirs, coal seams, landfill gas transport, etc (Huang et al. 2013; Zhang et al. 2011; Xue et al. 2005). However, the nonlinear of the hydraulic parameters in parameter dynamic change and the different soil properties were merely considered in those studies. The objective of this paper was to develop a mathematical model which considered the nonlinear of the hydraulic parameters including the variable porosity and permeability and to study the behavior of coupling model of water pore pressure and soil stress. A general Galerkin finite element method was used to solve the numerical solution of coupling model. Finally, the mathematical model was applied to 2D numerical model experiment with different soil properties to show the effects of nonlinear character of hydraulic properties on consolidation processes.

Governing equations

A general mathematical model describing the transient flow equation and the equilibrium equation can be derived based on the following assumptions: (1) The medium is an isotropic pore-elastic continuum; (2) fluid flow follows the Darcy's law; (3) the deformation of solid matrix abides by Terzaghi's (1943) law of effective stress

$$\sigma_{ij} = \sigma'_{ij} + p\delta_{ij}, \quad (1)$$

where σ_{ij} is the tensor of total stress, σ'_{ij} is the tensor of effective stress, and p is pore pressure, and δ_{ij} is Kronecker sign.

Equilibrium equation of stress (neglecting the body forces)

$$\sigma_{ij,j} = 0. \quad (2)$$

Geometrical equation

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (3)$$

Constitutive relation of strain and stress

$$\sigma'_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_v\delta_{ij}, \quad (4)$$

where $G = E/2(1+\nu)$ is the shear modulus of the porous medium, E is Yang's modulus, ν is Poisson's ratio, $\lambda = \frac{2G}{1-2\nu}$ is Lamé constant, and $\varepsilon_v = \partial u_x/\partial x + \partial u_y/\partial y + \partial u_z/\partial z$ is volumetric strain.

The equilibrium equation of rock described with displacement may be obtained by combining Eqs. (1–4)

$$Gu_{i,jj} + (\lambda + G)\varepsilon_{v,i} + p_i = 0. \quad (5)$$

When groundwater is pumping, not only the fluid flow has seepage velocity, but also solid granules have seepage velocity. So, the seepage velocity of fluid flow can be described as

$$V_f = V_r + V_s, \quad (6)$$

where V_f is the absolute velocity of fluid flow, and V_s is the absolute velocity of solid granules,

$$\nabla V_s = \nabla \left(\frac{\partial u}{\partial t} \right) = \nabla \left(\frac{\partial \nabla u}{\partial t} \right) = \frac{\partial \varepsilon_v}{\partial t} \quad (7)$$

V_r is relative velocity of fluid flow to solid granules and can be described as

$$V_r = -\frac{k}{n\mu} \text{grad}(p), \quad (8)$$

where k is hydraulic conductivity, n is porosity, and μ is kinetic viscosity.

The hydraulic conductivity of the porous medium can be related to the porosity as (Kozeny 1927; Carman 1937)

$$\frac{k}{k_0} = \frac{1}{1 + \varepsilon_v} \left(1 + \frac{\varepsilon_v}{n_0} \right)^3, \quad (9)$$

where k_0 is the initial hydraulic conductivity, and n_0 is initial porosity.

The continue equation of porous media water flow can be described as

$$\nabla[\rho_f n V_f] + \frac{\partial[\rho_f n]}{\partial t} = 0 \quad (10)$$

The continue equation of solid granules can be described

$$\nabla[\rho_s(1-n)V_s] + \frac{\partial[\rho_s(1-n)]}{\partial t} = 0 \quad (11)$$

It can be obtained by simplifying (10) and (11)

$$\rho_s(1-n)\nabla V_s + (1-n)\frac{\partial \rho_s}{\partial t} - \rho_s \frac{\partial n}{\partial t} = 0 \quad (12)$$

$$\rho_f n \nabla V_r + n \rho_f \nabla V_s + n \frac{\partial \rho_f}{\partial t} + \rho_f \frac{\partial n}{\partial t} = 0 \quad (13)$$

We can obtain by dividing ρ_f and ρ_s , and then add (12) and (13)

$$n \nabla V_r + \nabla V_s + \frac{n}{\rho_f} \frac{\partial \rho_f}{\partial t} + \frac{(1-n)}{\rho_s} \frac{\partial \rho_s}{\partial t} = 0 \quad (14)$$

The state equation of fluid can be described in equal temperature as (Li et al. 2003)

$$\rho_f = \rho_0 e^{[\beta_f(p-p_0)]} \quad (15)$$

We can also obtain the state equation of solid granules as

$$\rho_s = \rho_0 e^{[\beta_s(p-p_0)]} \quad (16)$$

We can obtain equation by integrate t as

$$\frac{1}{\rho_f} \frac{\partial \rho_f}{\partial t} = \beta_f \frac{\partial p}{\partial t} \quad (17)$$

The same as solid granules

$$\frac{1}{\rho_s} \frac{\partial \rho_s}{\partial t} = \beta_s \frac{\partial p}{\partial t}, \quad (18)$$

where β_f is the compressibility coefficient of porosity flow, and β_s is the compressibility coefficient of solid granules.

We can obtain equation by combining Eqs. (14–17)

$$\nabla \left(-\frac{k}{\mu} \nabla p \right) + \frac{\partial \varepsilon_v}{\partial t} + (\beta_s(1-n) + n\beta_f) \frac{\partial p}{\partial t} = 0 \quad (19)$$

In the case of a porous medium, the change in the porosity due to the change of water pore pressure and soil stress can be expressed by (7), (12) and (18)

$$\frac{\partial n}{\partial t} = (1-n) \left(\frac{\partial \varepsilon_v}{\partial t} + \beta_s \frac{\partial p}{\partial t} \right). \quad (20)$$

The mathematical model of fluid–solid interaction could be described by four nonlinear partial differential equations (5), (9), (19) and (20) with four dependent variables, p , u , k and n .

Finite element approximations

Equations (5), (9), (19) and (20) constitute a general mathematical statement of the physical problem of land subsidence due to pressure decline. Analytical solutions for this general system are not available except for simplified cases. Therefore, numerical approaches may be used to solve general solutions for this problem. The Galerkin finite element method (Thomée 2006) is chosen here owing to its ability to treat compound and complex geometries. In finite element method, a trial solution of the dependent variable ϕ , which represents p , u_x , u_y and u_z can be expressed by the basic functions as

$$\phi \approx \hat{\phi} = \sum_L^N \phi_L(t) N_L(x, y, z), \quad (21)$$

where $N_L(x, y, z)$ is the basis function, $\phi_L(t)$ is the value of the unknown variable, and N is the total number of nodes.

Applying the Galerkin's approach and Green's theorem to Eqs. (5) and (19) yields

$$\int_R \left[-\nabla N_L \frac{K}{\mu} \nabla (P - G_s) + N \frac{\partial \varepsilon_v}{\partial t} + N_L (\beta_s (1 - n) + n \beta_f) \frac{\partial P}{\partial t} \right] dR \\ = - \int_B N_L \frac{K}{\mu} \nabla P \cdot n dB \quad (22)$$

$$\int_R \left[G \left(\frac{\partial u_i}{\partial x_i} \frac{\partial N_L}{\partial x_i} + \frac{\partial u_j}{\partial x_j} \frac{\partial N_L}{\partial x_j} \right) + (\lambda + G) \frac{\partial N_L}{\partial x_j} \varepsilon_v + \alpha \frac{\partial N_L}{\partial x_j} P \right] dR \\ = \int_B N_L [(u_i + u_j) + (\lambda + G) \varepsilon_v + \alpha P] \cdot n dB, \quad i, j = x, y, z \quad (23)$$

where R is the region of interest with boundary B , and n is the outward unit vector.

Initial values of pressure head and displacements must be prescribed in the entire region of interest R . The boundary conditions associated with equation are of the following types (Huyakorn et al. 1986): Dirichlet type used in pressure head and displacement; Neumann type used in normal flux to Darcy velocity, to solve the nonlinear problem associated with changes in the hydraulic properties due to pore water pressure decreases and solid matrix deformation at each time step, for which the incremental Picard scheme is adopted. For the next iteration level, the hydraulic properties are updated to compute the coefficient matrix and the dependent variables.

Model verification

A simplified case with known analytical solutions is utilized to verify the finite element model. In this model verification example, a fully penetrating well of radius pumping at a constant rate from a saturated confined aquifer, the aquifer is assumed to be homogeneous and isotropic in space and time. The analytical solutions are derived by Bear and Corapcioglu as follow

$$s = -\frac{Q_w}{4\pi T} W(u) \quad u = \frac{r^2}{4C_v} = \frac{Sr^2}{4Tt} \quad (24)$$

$$\Delta z = -\frac{Q_w}{8\pi C_v} W(u) \quad (25)$$

$$U_r = -\frac{Q_w t}{4\pi r B} [u W(u) + 1 - e^{-u}] \quad (26)$$

where s is the drawdown, Δz is the vertical displacement at the top surface, U_r is the horizontal displacement, $W(u) = \int_u^\infty \frac{e^{-x}}{x} dx$ is the well function. $T = KB$ is the transmissivity,

S is the aquifer's storativity, Q_w is pumping rate, and $C_v = T/S$ is the consolidation coefficient of the confined aquifer.

As a numerical example, the hydraulic properties of the confined aquifer are given as follows: The initial hydraulic head is assigned to be equal to 0 everywhere in model. A confined aquifer with $K = 6.69 \times 10^{-5}$ m/s, $B = 142$ m, $Q_w = 0.5$ m³/s, $C_v = 6 \times 10^3$ cm²/s.

As shown in Fig. 1, the numerical solutions are demonstrated by circles. Analytical solutions for the drawdown, horizontal and vertical displacements are indicated by solid lines. The relationships of the drawdown, displacement versus the distance from the well after continuous pumping for 3 years are shown in Fig. 1a. The relationship of the drawdown and displacement versus time at the distance of 3 km from the pumping well is shown in Fig. 1b. Results show that the simulated drawdown and displacements match well with those of the analytical solutions.

Model application

The proposed model is applied to an ideal three layers numerical experiment. The purpose of the numerical experiments is to analyze the nonlinear effects of hydraulic properties in the seepage and consolidation coupling phenomena. For such a case, the initial pressure head in numerical model is set equal to 0 m. The land surface, i.e., the top boundary, is set free to move both vertically and horizontally. At the bottom boundary, the soil is restrained from both horizontal and vertical movements. The no-flow boundary condition is assigned on the right side and bottom side of the model region. The initial displacement is equal to 0 m throughout the model. The boundary displacements in the x direction on the two sides located at $x = 0$ and $x = 1,000$ are constrained. A 90-m-thick three layers aquifer system with a radial extent of 1,000 m is considered in this case as shown in Fig. 2. The first aquifer system is a 30-m-thick phreatic aquifer, and the middle aquifer system is a 10-m-thick aquitard aquifer, and the third layer is a 50-m-thick confined aquifer. The pumping well is drilled vertically through the confined aquifer and the pumping rate is set to $-6 \times t/360$ at $x = 0$. The hydraulic properties parameters of three aquifers are obtained from these references (Kim and Parizek 2005; Biot 1955; Leake and Hsieh 1997; Wang 2000; Hsieh 2006) and used in model as shown in Table 1.

The trend of porosity and hydraulic conductivity around the pumping well at the confined aquifer is shown in Fig. 3. The hydraulic conductivity and porosity dramatically decrease along time and slightly decrease along distance from pumping well. After 10 years of pumping, the hydraulic conductivity with NHP model is estimated to be 0.88k0, and the porosity with NHP model is about 0.95n0.

Fig. 1 Spatial distributions (a left) and temporal change (b right) of drawdown, vertical displacement and horizontal displacement in the confined aquifer

Fig. 2 Constant condition of numerical coupling model

Table 1 Basic parameters

Parameters	Phreatic aquifer	Aquitard	Confined aquifer
Initial porosity n_0 (dimensionless)	0.4	0.5	0.3
Viscosity coefficient μ (MPa)	2×10^4	1.5×10^4	1.5×10^4
Acceleration of gravity g (m/s^2)	9.81	9.81	9.81
Elastic modulus E (MPa)	3.6	2.6	3.6
Shear modulus G (MPa)	1.44	0.96	1.44
Poisson's ratio ν	0.25	0.35	0.25
Lame constant λ (MPa)	1.724×10^4	1.624×10^4	1.824×10^4
The first biot modulus H (MPa)	3.42×10^4	2.42×10^4	3.82×10^4
The second biot modulus R (MPa)	2.35×10^4	1.35×10^4	2.95×10^4
Initial hydraulic conductivity k_0 [m/d]	25	0.01	35
Initial pressure p_0 (MPa)	1.0	1.0	1.0
The compressibility of water (GPa^{-1})	0.5	0.5	0.4
The compressibility of soil (MPa^{-1})	0.3	0.2	0.25

The nonlinear hydraulic properties (NHP) are considered in coupling model and the numerical solutions with and without the NHP are shown in Figs. 4, 5 and 6. The result of pressure head with the NHP significantly increases

near the pumping well, but slightly decreases from the distance of pumping well compared to the numerical solutions without the NHP model. After 10 years of pumping, the pressure head dropped from -35 to -50 m

Fig. 3 The special distribution of hydraulic conductivity (*left*) and porosity (*right*) at the confined aquifer

Fig. 4 The distribution of head with and without NHP model at the confined aquifer

with NHP model, but it did from -33.6 to -50 m without NHP model. The drawdown is 15 m with NHP model, whereas it is 16.4 m without NHP model. Pressure head at $r = 500$ m is about -38.4 m with the NHP model, whereas about -37.3 m without NHP model. The vertical and horizontal displacements are larger with NHP model than without NHP model. The vertical displacement at $r = 500$ m is about -1.57 cm with NHP model, but about -1.47 m without NHP model after 10 years pumping. The horizontal displacement at $r = 200$ m is about -8.14 m with NHP model, but about -7.68 m without NHP model after 10 years pumping. The reason is that the water level decline due to groundwater withdrawal induces the soil compression and the porosity and permeability decrease. Decrease of permeability in regions adjacent to the

Fig. 5 The distribution of horizontal displacement with and without NHP model at the confined aquifer

Fig. 6 The distribution of vertical displacement with and without NHP model at the confined aquifer

Fig. 7 The contour distributions of horizontal displacement without (*dashed lines*) and with NHP (*solid lines*) model at the profile

Fig. 8 The contour distributions of vertical displacement without (*dashed lines*) and with NHP (*solid lines*) model at the profile

pumping well produces hydraulic gradient and seepage forces that may result in accelerated subsidence. Therefore, the effect of the NHP is an interaction process of pore water pressure and soil consolidation deformation and should not be ignored.

The contour distributions of horizontal and vertical displacements after 10 years of pumping are shown in Figs. 7 and 8. The solid lines stand for the displacement with NHP model, and the dashed lines describe the displacement without NHP model. The max horizontal displacement shows up at about $r = 200$ m from the pumping well and after that the horizontal displacement went small and the value is 0 at $r = 1,000$ m. The reason is that the boundary of the model is restricted and cannot take place lateral deformation. The displacement around pumping well is obvious and the value is smaller along distance from pumping well. The horizontal

displacements change dramatically in the aquitard than the other two aquifers. The peak value is reached around $r = 250$ m from pumping well in the phreatic aquifer, whereas its value is decreased along vertical depth. The maximum of horizontal displacement is easy to achieve near pumping well. After 10 years of pumping, the horizontal displacement is 12.94 cm at (200, -20) with NHP model, but the value is 11.93 cm at (200, -20) without NHP model. The vertical displacement is 4.82 cm at (200, -20) with NHP model, but the value is 4.7 cm at (200, -20) without NHP model. As shown in Fig. 8, the value of vertical displacement is from 0 to 2.5 cm in 50-m-thick confined aquifer, but it is from 2.5 to 4.5 cm in 10-m-thick aquitard. The reason is that the clay soil layer releases water when pumping in the confined aquifer, but it is hard to be supplied because of its weak permeability.

Conclusions

This study considered the characteristics of nonlinear changes in the porosity and the hydraulic conductivity that could occur in the field. The deformations of soil were associated with the water pressure change. To show the effect of nonlinear of hydraulic properties, a mathematical model employing the Galerkin finite element method had been proposed to simulate the deformation dependencies of hydraulic properties due to the water pressure decrease in aquifers. The numerical model was verified by comparing with the analytical solutions in the confined aquifer. Results showed that the simulated drawdown and displacements matched well with those of the analytical solutions. The proposed model was applied to an ideal three layers numerical experiment to analyze the NHP in different soils. The result of pressure head with the NHP significantly increased near the pumping well, but slightly decreased from the distance of pumping well compared to the numerical solutions without the NHP model. The value of displacement was smaller without NHP model than with NHP model. The displacement changed dramatically in the aquitard than the other two aquifers. The effect of the NHP had its own importance not only in the groundwater flow field, but also in the solid skeleton deformation field, and hence it is obvious and cannot be ignored, especially when the long-term pore water pressure change and soil deformation induced by groundwater pumping. The numerical study demonstrated that the NHP character was more desirable than conventional groundwater theory for solving fully coupled soil–water phenomena associated with fluid flow through deforming media. Further studies of various hydrogeology problems are recommended to consider the heterogeneity concerning different stratigraphic conditions in the field.

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