Probabilistic modeling of rockfall trajectories: a review

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Abstract Probabilistic approaches are widely adopted for the modeling of rockfall trajectories, but are not widely discussed in the literature. This paper aims to help fill this gap by reviewing probabilistic models of rockfall trajectories, while providing some perspectives for future study. We first make it clear that, from a theoretical point of view, the probabilistic approach is necessitated by both the ontic (inherent) uncertainty associated with rockfalls and the epistemic (information) uncertainty associated with numerical modeling. The review suggests that there may be the potential to improve the probabilistic modeling of rockfall trajectories in various aspects, including the systematic probabilistic modeling criterion, the random sampling approaches employed for probabilistic variables, the probabilistic modeling of rock shape, and the probabilistic prediction of rockfall intensity. However, there are still some open questions regarding the promotion of probabilistic modeling in practice. It is not clear whether probabilistically treating all of the variables of rockfall trajectory model with reasoned distributions will lead to significantly improved results, or whether the improvements will be great enough (given the difficulties and costs involved) that it is worth quantifying all of the uncertainties involved in rockfall trajectory modeling. The answers to these

questions can be found in the practice of probabilistic modeling itself.

Keywords Rockfall trajectory · Probabilistic modeling · Uncertainty · Random sampling · Rock shape · Rockfall intensity

Introduction

Rockfall is a phenomenon in which relatively small volumes of rock rapidly tumble down a slope. What makes a rockfall distinct from a rockslide and a rock avalanche is the independent movements of individual fragments (e.g., Hungr and Evans 1988). Numerical simulations predict either the runout zones of rockfalls based on empirical observations (e.g., Jaboyedoff and Labiouse 2011) or the trajectories of individual rocks using Newtonian mechanics (see, e.g., “[Review of probabilistic modeling](#)”). Trajectory modeling has the advantage of representing the physics of rockfalls and, in practice, it also stands out due to its ability to predict the bounce heights and kinetic energies of moving rocks, which are particularly important in the design of countermeasures (e.g., Agliardi et al. 2009; Pantelidis and Kokkalis 2011; Bourrier et al. 2015).

A lot of models have been developed to simulate rockfall trajectories. They can be grouped into three categories based on their terrain and simulation properties. First, the slope topography can be represented by either a 2D profile or a 3D terrain. Second, the kinematics of rockfalls can be modeled using a lumped mass method, a rigid body method, a hybrid method combining both the lumped mass method and the rigid body method, or a discrete element method. Third, rockfall trajectories can be simulated using either a deterministic approach or a

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probabilistic approach. In a probabilistic approach (or stochastic approach), rockfall trajectories are determined by model inputs that vary randomly within predefined ranges according to certain probability distributions. To address the inevitable uncertainties associated with rockfall modeling, most rockfall trajectory models adopt a probabilistic rather than a deterministic approach (e.g., Volkwein et al. 2011; Turner and Duffy 2012). In practice, considering the uncertainties involved in rockfall modeling as well as computational efficiency, probabilistic modeling using simplified algorithms is often applicable (Turner and Duffy 2012).

The authors, however, are not aware of a major paper that specifically discusses the probabilistic modeling of rockfall trajectories, even though this approach has been widely adopted. This paper provides a review of the probabilistic modeling components in various rockfall trajectory models, followed by some perspectives for future study. We begin with the fundamental question of why the probabilistic approach is necessary to simulate rockfall trajectories from a theoretical point of view, and end with a discussion of some open questions concerning probabilistic modeling in practice.

Why probabilistic modeling is necessary

Inherent uncertainty of rockfalls

At a typical rockfall site, field investigations usually find several individual rocks scattered on the slope (Fig. 1). An instinctive question about this phenomenon is: why do different rocks come to rest at different locations on the

same slope? From a deterministic point of view, the trajectory of a rockfall depends on (1) where the rock detached from, (2) which rock is detached, (3) how the rock detached, and (4) the properties of the slope. The first three factors correspond to the location of the source (i.e., planar coordinates and height), the properties of the rock (e.g., material, structure, size, and shape), and the initial conditions (e.g., initial velocity), respectively. The slope properties include topography, roughness, material, and land cover, among others. The scattering of rockfall trajectories across a slope is not surprising, as rockfall dynamics are defined by many variable factors. Variations occur in the source location, rock properties, initial conditions, and slope properties between different rockfall events, thus creating an inherent uncertainty in rockfall phenomena. The detachment of different rock blocks from different locations on different slopes with different initial conditions will result in different rockfall trajectories. Experiments reveal that minor variations in the initial conditions can result in significant scattering of rockfall trajectories, even if the same rock sample is used (Lan et al. 2014). Therefore, it is the inherent uncertainty (stochasticity) of the source location, rock properties, initial conditions, and slope properties that accounts for the scattering of rockfall trajectories and necessitates probabilistic modeling.

Uncertainties in numerical modeling

Rockfall dynamics include the free fall of rock, the rock–slope interactions, the rock–vegetation interactions, and often the fragmentation of rock. The dynamics of free fall are clear and can be predicted with analytical models with

Fig. 1 A photo of a typical rockfall site in Dongchuan, Yunnan, China shows several individual rocks scattered across the slope. The fact that different rocks detached from different locations with different initial conditions accounts for the scatter in rockfall trajectories, and implies the inherent stochasticity of rockfalls

quite high precision if the influence of the air is ignored. Contrarily, the rock–slope interactions are extremely complicated, and include the impact, rebound, rolling, and sliding of rock on the slope. Taking the impact and rebound of rock as an example, the time functions of interaction forces during this process are extremely complex and will probably never be determined precisely. Any model describing rock–slope interactions relies on some degree of simplification (Bourrier and Hungr 2011). Errors in modeling can derive from either the model or the model inputs. The errors in the model can be associated with the physics or the solution of the model; in other words, either the mathematical equations (usually differential equations) describing the physics or the solutions of these equations can be imprecise. Even when both the physics and the solution of the model are exact, model input errors are still inescapable. Field data from a rockfall site—such as terrain conditions and initial conditions—are used as model inputs, but can be insufficient or inaccurate simply due to time, cost, and access constraints. In dynamic systems, propagations of imperceptible differences may result in considerably divergent states (e.g., Lorenz 1963). Similarly, the inevitable inaccuracy of model inputs will lead to significant uncertainties in the modeling of rockfall trajectories.

Rebound behavior is constrained by the substrate properties, rock properties, and incident configurations (e.g., Labiouse and Heidenreich 2009; Bourrier and Hungr 2011). In numerical modeling, errors in the classification of substrate and rock properties are inevitable. For example, any representation of slope topography or rock shape will be a simplification of the reality. These errors will introduce uncertainties into the simulation of rebound processes. In addition, since the final states (e.g., translational and rotational velocities) of an impact constrain the initial conditions of the next impact, the uncertainties in the solution of an impact will lead to errors in the predictions of the impact location and incident configurations of the next impact, which will in turn affect the solutions for all of the subsequent impacts. As a result, significant uncertainties can evolve from minor errors, no matter how sophisticated the model is. Theoretically, parameter calibration does not help with this complication, since minor errors must still exist in the modeling of future rockfall events even when the parameters are calibrated with historical events. Therefore, a probabilistic approach is needed, especially when simulating the interactions between rock and slope, since uncertainties in numerical modeling are inevitable.

Two types of uncertainty

There are generally two types of uncertainty: ontic and epistemic (e.g., van Asselt and Rotmans 2002). Ontic

uncertainty, also called inherent or stochastic uncertainty, derives from the inherent variability of the properties of complex objects. Epistemic uncertainty, also called information uncertainty, derives from the limited knowledge of the properties of complex objects. The epistemic uncertainty can be moderated by gathering more complete and precise knowledge. Contrarily, ontic uncertainty is an inherent characteristic of complex phenomena and thus cannot be eliminated by developing a better understanding of or improved analytical techniques for the processes and entities involved. The uncertainties accompanying rockfall trajectory modeling are shown in Fig. 2. The inherent stochasticity of rockfalls corresponds to the ontic uncertainty. On the other hand, since the knowledge of rockfall processes and individual rockfall events is limited, epistemic uncertainty is automatically incorporated when people try to model rockfall trajectories. It is thus clear that, from a theoretical point of view, it is both the ontic uncertainty associated with rockfalls and the epistemic uncertainty accompanying numerical modeling that make a probabilistic approach necessary when simulating rockfall trajectories.

Review of probabilistic modeling

Existing rockfall models have already been reviewed in some recent publications (e.g., Volkwein et al. 2011; Dorren et al. 2011; Frattini et al. 2012; Turner and Duffy 2012). This section specifies how various models implement the probabilistic modeling of rockfall trajectories, with more detailed and additional information presented. Models simulating rockfall trajectories along a 2D profile and on a 3D terrain are introduced separately. Typical and widely used models are illustrated first, followed by brief introductions to some other models. It must be emphasized that this review is not exhaustive. With regard to the models known to the profession, especially for the models developed several decades ago, direct access to the program and relevant publications is limited. Information about some commercial software packages is also limited due to commercialization issues. The features of the models presented here are based on descriptions given in the relevant documentation if the program (software) is not available.

2D models

CRSP

CRSP (Colorado Rockfall Simulation Program) was originally developed by Pfeiffer (1989). The algorithm of CRSP did not change until version 5.0 (Bartingale et al.

Fig. 2 Uncertainties present in the modeling of rockfall trajectories

2009). In CRSP, the surface roughness is defined as the maximum deviation perpendicular to the slope within a certain distance. The impact angle is related to the local slope angle. At each impact, the random variation in the surface roughness results in random variation in the slope angle and random variation in the impact angle. The surface roughness is a function of rock size and is determined by field observations of the specific rock radius in early versions. An important improvement in version 5.0 is that the surface roughness is adjusted according to the rock size in the algorithm. Therefore, one input value of surface roughness can be adopted by different rock diameters in a single simulation. However, the random variation in impact angle is largely responsible for the statistical variation in rockfalls modeled by CRSP.

RocFall

RocFall was originally developed by Stevens (1998). The latest commercially available version is RocFall 5.0 (Rocscience Inc. 2013). RocFall is a statistics-based software program. Almost all the inputs of RocFall can be varied randomly. For each rockfall event, the source location, rock properties, and initial conditions can be defined probabilistically. The rock can be released from a randomly selected location on a “line seeder.” The mass and the density of the thrown rock can be randomly sampled from predefined distributions, including normal, uniform, triangular, beta, exponential, lognormal, and gamma distributions. The shape of the rock can be randomly selected from a library of shapes. The initial conditions (i.e., the horizontal velocity, the vertical velocity, the rotational velocity, and the initial rotation) can also be

randomly sampled from predefined distributions. After the rock has been thrown, the interactions between the rock and the slope can also be a stochastic process. The material properties of each slope segment (i.e., the normal restitution, the tangential restitution, the dry friction, the rolling resistance, and the spacing and amplitude defining the slope roughness) can be randomly drawn from predefined distributions. One unique feature of RocFall is that the uncertainties in the representation of the slope are addressed. The coordinates of the vertices of the 2D slope profile are assumed to obey normal distributions, with zero standard deviation indicating that the slope profile is deterministic in the modeling. Another notable feature is the “pseudo-random” option, which allows the random number generator to be initialized with specific seeds. The fact that the same random seed yields identical random numbers makes the simulation results reproducible.

Other 2D models

There are many other 2D models of rockfall, but most are rarely used nowadays. SASS (Bozzolo and Pamini 1986) implements a probabilistic approach by starting rocks in random positions within a predefined area with random initial and boundary conditions, and randomly varying the local slope angles within predefined ranges. In a probabilistic block propagation model, the shock restitution coefficient and the initial rebound angle are treated as random variables (Paronuzzi 1989). In GeoFall (Ashfield 2001), pseudo-random values that have a uniform probability are used to vary the values of the coefficients of restitution and the slope angle. In Rockfall (Dr. Spang GmbH 2008), some parameters can be varied randomly within predefined

ranges, including the dynamic and static frictions, the normal and tangential damping coefficients (restitution coefficients), the rolling resistance, and the radius of trees if the effects of vegetation are incorporated. NURock (Spadari et al. 2013) randomly varies the local slope angle in the same manner as CRSP, and has an optional random variation for the relationship between the normal coefficient of restitution and impact angle. In a program that incorporated 2D discontinuous deformation analysis (DDA) (Ma et al. 2013), a probabilistic approach is applied to the friction angle, impact damping coefficient, and tree resistance coefficient.

3D models

STONE and Hy-STONE

STONE was first presented by Guzzetti et al. (2002). It adopts a lumped mass method. In STONE, the stochasticity of the source location is addressed by inputting a raster map defining the source cells and the local frequency (number) of detached rocks. Rocks are released horizontally. Although the horizontal launch velocity is constant, the horizontal launch direction is drawn randomly from a stochastic range for each rock. Similarly, the coefficients of restitution and friction are also drawn randomly from predefined ranges. An updated version of this program, Hy-STONE, incorporates a hybrid (mixed kinematic and dynamic) algorithm and additional capabilities for probabilistic analyses (Crosta and Agliardi 2004). Hy-STONE addresses the uncertain nature of rockfall processes partly by randomly sampling most parameters from different probability density distributions. From the available literature, it is not clear exactly which of the parameters can be varied randomly. One advantage of Hy-STONE is that it can repeat the simulation through a pseudo-random approach. The literature states that blocks with different shapes and masses can be released from each source point (Frattini et al. 2008), and although the random sampling of block sizes is discussed (e.g., Agliardi and Crosta 2014; Frattini et al. 2014), the random sampling of shapes is not. Hy-STONE can also simulate the effects of vegetation and fragmentation using a probabilistic approach (Frattini et al. 2012). The distribution of fragment sizes and the kinetic energy of each generated fragment are determined based on user-defined distribution constraints.

Rockfall Analyst

Rockfall Analyst (Lan et al. 2007, 2010) simulates rockfall trajectories using a lumped mass model. It is unique in that it is an extension of ArcGIS. Probabilistic modeling of rockfalls is implemented by simulating a large number of

trajectories. Several seeders can be uniformly sampled along a polyline according to a predefined sampling distance. Several rocks can be launched in different directions from each seeder. The default start direction points to the dip direction of the cell in which the seeder is located, while other launch directions are determined by a predefined angle interval. No randomness is incorporated into the code, so all simulations performed with the same model inputs will be identical. The interactions between the rock and slope are fully described by three material properties: the coefficients of normal restitution, tangential restitution, and friction. All three parameters are constant during simulations. The mass and shape of the block does not influence the rockfall dynamics, as the model employs a lumped-mass approach. Only the inherent stochasticity of the source location and the initial direction of rockfall phenomena are considered in Rockfall Analyst.

Rockyfor3D

The most recent version of Rockyfor3D is 5.1 (Dorren 2012). This program has been used widely to evaluate rockfall hazards in forested areas (e.g., Radtke et al. 2014). Several inputs of this model are randomly varied. The three dimensions of rock blocks are varied randomly by a predefined percentage for each simulation. The slope surface roughness is represented by an obstacle height (MOH). For each cell, three representative values of MOH are predetermined by the user. The probabilities that these three representative MOH values will be encountered by a falling rock are 70, 20, and 10 %, respectively. During each rebound calculation, the MOH value in a cell is randomly chosen from the three representative values according to their probabilities of occurrence. In order to represent the variance in surface roughness observed in nature, the value of the calculated tangential restitution coefficient is also varied randomly by ± 10 % before it is used in the calculation of the tangential rebound velocity. Similar to CRSP, the slope angle at the impact location is decreased randomly during each rebound, while the maximum decrease in the slope angle is fixed at 4° . The program also considers the deviation of the rebound direction from the theoretical value determined by the incident direction and the slope aspect. The probabilities for the ranges of variation of these deviation angles are estimated based on field experiments. The deviation of the angle of fall direction following impact with the slope surface or vegetation is selected randomly according to their probabilities of occurrence.

PICUS Rock'n'Roll

PICUS Rock'n'Roll consists of a 3D rockfall module embedded in a patch-based forest simulator (Woltjer et al.

2008; Rammer et al. 2010). Rock trajectories can start either from predefined or from randomly selected positions within the simulation area. The coefficients of restitution are varied within a specified range. Stochastic components are assigned to the lateral deviation after tree impacts, as well as to the lateral deviation and the jump angle after ground impacts. The perpendicular distance between the incident direction of the rock and the tree center is determined by randomly selecting a value within the radius of the stem. Another unique feature of this tool is the stopping algorithm (Rammer et al. 2010). Experiments with virtual surfaces of block deposits were designed to develop a relationship between the stopping probability and the combination of surface type and falling rock size. For each ground impact, the stopping probability is estimated using this relationship, and this parameter determines whether a rock stops, together with a uniform random number.

RocPro3D

RocPro3D (RocPro3D 2014) evolved from Pir3D (Cottaz and Faure 2008). Pir3D takes a lumped mass approach, while RocPro3D also incorporates a rigid body approach. Three block parameters can be represented by probabilistic (uniform) variables, namely the starting position, the mass (and size), and the starting conditions (velocity or falling height). The five soil parameters can be represented by probabilistic (uniform or Gaussian) variables, namely the dynamic friction coefficient, the normal and tangential restitution coefficients, as well as the lateral (horizontal) deviation and the flattening (vertical) of the rebound angle. The starting point of each trajectory is randomly chosen along seeder lines. An option of pseudo-random variation is also available for repeating the simulations. One unique feature of this model is that it allows larger uncertainties at low velocities than at high velocities to be taken into account. A maximum variability is assigned to zero velocity and a residual variability is used above a threshold velocity. Between zero velocity and the threshold velocity, the varied value is selected from a linear function.

Other 3D models

The advantage that 3D models have over 2D models is obvious. Advances in the computing power of personal computers have allowed several 3D rockfall models to be developed. Dudt and Heidenreich (2001) proposed EBOULEMENT, a program to treat the uncertainty in a 3D numerical rockfall model. The original literature is not accessible; however, according to Turner and Duffy (2012), this program varies the rock dimensions, the fall height, the initial Y coordinate, as well as the expected values of the normal restitution coefficient, the plastic

factor, and the friction coefficient for each new simulation of a trajectory. During the simulation of a specific trajectory, this program also randomly varies the normal restitution coefficient, the plastic factor, and the friction coefficient around their expected values, and the roughness angle within a predefined range. Similar to Rockfall Analyst, ROTOMAP (Scioldo 2006) implements a probabilistic approach by simulating a large number of trajectories. The total number of trajectories is the product of the number of starting points, the number of initial velocities and the number of initial launch directions. CRSP-3D (Andrew et al. 2012) uses a DEM (discrete element method) to simulate rockfall trajectories. It randomly chooses the rock size, slope roughness, and slope hardness within designated ranges. For each trajectory, the rockfall starting location is also randomly chosen within a predefined release zone. The rockfall module of RAMMS is a sophisticated model that takes rock shape into account and adopts the methods of multi-body dynamics and non-smooth contact dynamics (Leine et al. 2014). Statistical variability in rockfall trajectories can be obtained by varying release position and block orientation.

Perspectives for future work

The probabilistic components in various rockfall trajectory models are presented in Table 1. A component is marked as probabilistic if any parameter in that component adopts a probabilistic approach. In the following paragraphs, we discuss aspects of probabilistic modeling which have the potential for improvement and which should be concentrated on in future work.

Systematic probabilistic modeling

The simulation of rockfall trajectories is complicated by both the inherent stochasticity of rockfalls in nature and the epistemic uncertainty associated with numerical modeling. The inherent stochasticity can be managed by probabilistically modeling the source location, rock properties, initial conditions, and slope properties, while the epistemic uncertainty can be managed by probabilistically modeling the dynamics of rockfalls, including rock–slope interactions, rock–vegetation interactions, and the fragmentation of rock. Systematic probabilistic modeling of rockfall trajectories should involve all of the aspects of inherent stochasticity and rockfall dynamics. Thus, an intuitive approach is to adopt a probabilistic approach for all of the components of rockfall trajectory models. However, the range of results will become too wide if all of the components are treated probabilistically. We will discuss this issue in “Probabilistic modeling in practice.”

Table 1 Probabilistic components of typical models for simulating rockfall trajectories

Name ^a	References ^b	D. ^c	Kinematics ^d	Probabilistic modeling ^e								Pseudo ^f
				Ontic uncertainty			Epistemic uncertainty					
				Source ^g	Rock ^h	Initial ⁱ	Slope ^j	Rebound ^k	Slide/ roll ^l	V. ^m	F. ⁿ	
CRSP	Pfeiffer (1989)	2D	Hybrid	○	○	○	●	●	○	×	×	○
RocFall	Rocscience Inc. (2013)	2D	Hybrid	●	●	●	●	●	●	×	×	●
SASS	Bozzolo and Pamini (1986)	2D	Hybrid	●	○	●	●	○	○	×	×	○
N.A.	Paronuzzi (1989)	2D	Hybrid	○	○	○	○	●	○	×	×	○
GeoFall	Ashfield (2001)	2D	Rigid body	○	○	○	●	●	○	×	×	●
Rockfall	Dr. Spang GmbH (2008)	2D	Hybrid	○	○	○	○	●	●	●	×	○
NURock	Spadari et al. (2013)	2D	Lumped mass	○	○	○	●	●	○	×	×	○
N.A.	Ma et al. (2013)	2D	DDA	○	○	○	○	●	●	●	×	○
STONE	Guzzetti et al. (2002)	3D	Lumped mass	●	○	●	○	●	●	×	×	○
Hy-STONE	Crosta and Agliardi (2004)	3D	Hybrid	●	●	●	○	●	●	●	●	●
Rockfall Analyst	Lan et al. (2007)	3D	Lumped mass	●	○	●	○	○	○	×	×	○
Rockyfor3D	Dorren (2012)	3D	Hybrid	○	●	○	●	●	×	●	×	○
PICUS Rock’n’Roll	Rammer et al. (2010)	3D	Hybrid	●	○	○	○	●	○	●	×	○
RocPro3D	RocPro3D (2014)	3D	Hybrid	●	●	●	○	●	●	×	×	●
EBOULEMENT	Dudt and Heidenreich (2001)	3D	Rigid body	●	●	○	●	●	●	×	×	○
ROTOMAP	Scioldo (2006)	3D	Lumped mass	●	○	●	○	○	○	×	×	○
CRSP-3D	Andrew et al. (2012)	3D	DEM	●	●	○	●	●	●	×	×	○
RAMMS::ROCKFALL	Leine et al. (2014)	3D	Rigid body	●	○	●	○	○	○	×	×	○

^a Names of rockfall trajectory models. *N.A.* not available

^b References in which information about rockfall trajectory models can be found

^c Dimensions in which rockfall trajectories are simulated

^d Method used to model the kinematics of rockfall. *DDA* and *DEM* indicate discontinuous deformation analysis and discrete element method, respectively

^e Components of rockfall trajectory models that adopt a probabilistic approach are marked as follows: ● = probabilistic, ○ = deterministic, × = does not have this component

^f Availability of option of a pseudo-random approach that makes the simulations repeatable: ● = available, ○ = not available

^g Source location of rockfalls, including planar coordinates and release height

^h Rock properties, including size, shape, and density, etc

ⁱ Initial conditions, including initial velocity and launch direction, etc

^j Slope properties, including local slope angle and surface roughness, etc

^k Parameters of the impact and rebound model, including normal and tangential coefficients of restitution, etc

^l Parameters of the sliding and rolling model, including coefficient of friction and rolling resistance, etc

^m Parameters used to model rock–vegetation interactions

ⁿ Parameters used to model rock fragmentation

Although some models apply a probabilistic approach for most of their components, such as RocFall, Hy-STONE, and RocPro3D, most models do not fulfill the systematic probabilistic criterion. In addition, most models do not have modules for modeling rock–vegetation interactions or rock fragmentation (Fig. 3). Modules for rock fragmentation are especially rare. All of the existing modules for rock–vegetation interactions and rock fragmentation in the listed models use a probabilistic approach. This is not surprising, since rock–vegetation interactions and rock fragmentation are processes that can be just as complex as rock–slope interactions. More information on these processes can be found in the literature on rock–vegetation interactions (e.g., Dorren et al. 2007) and rock fragmentation (e.g., Giacomini et al. 2009; Wang and Tonon 2011). Although important, these processes do not occur in all rockfall events, and are therefore not considered mandatory for rockfall modeling.

The most common element that is treated probabilistically in rockfall modeling is the rock rebound process. Only four of the 18 listed models do not apply a probabilistic approach to the rebound model (Fig. 3). The rock properties are least likely to be probabilistic. Only six out of the 18 models apply a probabilistic approach for rock properties (Fig. 3). Rock shape is difficult to define accurately. We discuss the representation of rock shape in more detail later in the text. It is, however, relatively easy to statistically quantify rock size. The size distribution of detached rocks can be approximated by performing field measurements of the in situ block size distribution (e.g., Sturzenegger et al. 2011; Elmouttie and Poropat 2012) or the size distribution of rockfall blocks from past events that have come to rest on the slope (e.g., Fityus et al. 2013). Despite this, size distributions of rocks are rarely used in the modeling of rockfall trajectories. Indeed, some case studies involving size distributions of rocks (e.g., Lambert

et al. 2012; Spadari et al. 2013; Macciotta et al. 2014) do not directly use size distributions for randomly sampling rock size in probabilistic modeling. This suggests that a systematic criterion is still not adopted by most probabilistic models of and modeling works on rockfall trajectories.

Random sampling approach

The most common method of sampling probabilistic parameters in trajectory modeling is to randomly select a value within a predefined range from a predefined distribution. The most common distribution used for random sampling is the uniform distribution. That is to say, most random variables within the probabilistic components of rockfall trajectory models are uniformly sampled. Uncertainty modeling of source location is special as it concerns the spatial domain. The rocks are launched from points sampled along starting lines (e.g., Rocscience Inc. 2013; Lan et al. 2007), or within release zones (e.g., Woltjer et al. 2008) in some models. Generally, these points are sampled spatially in a uniform manner. The normal (Gaussian) distribution is also frequently used. This trend prompts the question of why these particular distributions are used to randomly sample probabilistic parameters. In other words, do we have an adequate reason to assume that a probabilistic parameter obeys a certain form of probability distribution? The use of incorrect distributions of probabilistic parameters in rockfall trajectory models will result in inaccurate probabilistic analysis of the rockfall hazard. Therefore, the logical argument is that random sampling approaches for all probabilistic parameters should be based on empirical or theoretical knowledge.

Finding the distribution that a probabilistic parameter obeys will never be an easy task. Take the most common probabilistic rebound model as an example. Many models

Fig. 3 Frequencies of probabilistic rockfall trajectory models incorporating various model components. The total number of models taken into account is 18

randomly vary the coefficients of restitution within predefined ranges (e.g., Guzzetti et al. 2002). However, there is currently no evidence that the values of restitution coefficients in a range have equal probabilities or conform to other types of probability distribution. This is why only a few models randomly sample probabilistic parameters according to relevant theories or experiments. The results of field experiments and virtual experiments are used to determine the deviation angle probabilities after impact in Rockyfor3D (Dorren 2012) and the stopping probabilities in PICUS Rock'n'Roll (Rammer et al. 2010). In STONE (Guzzetti et al. 2002) and Hy-STONE (Crosta and Agliardi 2004), different numbers can be assigned to different source cells as a proxy for onset probability of rockfalls. Again, much work still needs to be done to apply susceptibility analysis (e.g., Fernandez-Hernandez et al. 2012; Valagussa et al. 2014) when characterizing the spatial variation of rockfall sources.

Probabilistic modeling of shape

The effects of rock shape on rockfall dynamics have been confirmed by both field observations (e.g., Haas et al. 2012) and numerical modeling (e.g., Glover et al. 2014; Zheng et al. 2014). Considering rock shape in the modeling of rockfall trajectories is not, however, a simple task. This is partly because it is difficult to both obtain precise data on rock shape and to precisely represent rock shape. The 3D shape of a rock is well represented in rigid body based models (Leine et al. 2014), and in discrete element modeling (e.g., Andrew et al. 2012; Zheng et al. 2014). The probabilistic modeling of rock shape is even more challenging than representing rock shape in numerical modeling. Unlike rock size, rock shape cannot be represented by a single quantity. The stochasticity of rock size can be quantified by a probability distribution. However, finding a probability distribution to characterize the natural randomness of rock shape is not straightforward.

There are some models that manage to implement a probabilistic approach for rock shape. The shape of a rock block is randomly selected from some predefined shapes in RocFall (Rocscience Inc. 2013), while the three dimensions of a rock can be randomly varied within predefined ranges in the Rockyfor3D program (Dorren 2012). However, the shapes that are possible in these models are only a small sample of the infinite set of rock shapes possible in nature (or at the source location). In addition, whether each possible shape in the model should have the same chance of being selected as the other shapes is an open question. A practical solution could be to first quantify the effects of shape on rockfall dynamics and then to find a probability distribution that fits. The effects of rock shape, for example on the rebound processes, can be implicitly included in

other quantities like the restitution coefficients, the deviation angle after impact, or the local slope perturbation angle (e.g., Bourrier et al. 2012). It is also recommended that a new quantity to exclusively account for the effects of rock shape should be introduced. However, whether the effects of rock shape are embedded in an existing quantity or in a new quantity, the science behind the decision should be clear, as a clear understanding is essential when determining an appropriate probability distribution, and is thus essential for the probabilistic modeling of rock shape.

Probability distribution of rockfall intensity

The probabilistic modeling of rockfall trajectories supports probabilistic hazard assessment by providing the spatial distribution of rockfalls and the probability distribution of rockfall intensity (e.g., Lan et al. 2013). Rockfall intensity can be quantified by potential energy or kinetic energy, and is crucial for risk assessment because it determines the degree of potential damage to the elements at risk. Mass and height determine the potential energy, and mass and velocity determine the kinetic energy. In practice, the calibration of model parameters is usually carried out by comparing the runout (spatial extension) of real rockfall events with that of simulated events (e.g., Agliardi and Crosta 2003; Holm and Jakob 2009), and only rarely by comparing rockfall intensity characteristics (e.g., Azzoni et al. 1995; Wyllie 2014). This is because both back-analysis of historical rockfall events (e.g., Paronuzzi 2009; Hürlimann et al. 2012) and in situ rockfall experiments (e.g., Dorren et al. 2006; Spadari et al. 2012) are laborious. It is much simpler for models to predict the runout than to predict the (bounce) height and velocity of a rockfall. Therefore, even if probabilistic modeling of rockfall trajectories can reasonably reproduce the spatial distribution of real rockfall events, reliable prediction of the probability distribution of rockfall intensity is not guaranteed.

We present a numerical experiment to show how different models yield similar rockfall runout zones but differing probability distributions of rockfall intensity. A typical section of the Canadian National Railway (CN Rail) was chosen as the study area (Lan et al. 2010). Fifty probabilistic rockfall trajectories starting from a single source point were simulated by both Rockfall Analyst (Lan et al. 2007) and RocPro3D (RocPro3D 2014). The inputs of the two models were adjusted so that they produced similar rockfall runouts, with most simulated trajectories ceasing on the railway track (Fig. 4). The two adjusted models produce different spatial distributions of rockfall bounce heights, for example over the railway track and during the initial stage of motion (Fig. 5). Frequency statistics showed that Rockfall Analyst predicts higher and more evenly distributed rockfall bounce heights over the railway track

Fig. 4 The probabilistic rockfall trajectories simulated by Rockfall Analyst (a) and RocPro3D (b). A LiDAR DEM with 1 m spatial resolution is used. As indicated by the *pink dashed ellipses*, most of the 50 probabilistic trajectories cease on the railway track for both

models. The major difference between the results for the two models is that the trajectories that cross the railway track are distributed on opposite sides

Fig. 5 The spatial distributions of rockfall bounce heights as modeled by Rockfall Analyst (a) and RocPro3D (b). Distinct differences are seen over the railway track (*pink dashed ellipses*) and during the initial stage of motion (*blue dashed ellipses*)

(Fig. 6). The probability that the rockfall bounce height over the railway track exceeds 1 m was estimated to be 7.2 and 0.5 % by Rockfall Analyst and RocPro3D,

respectively. This experiment suggests that predicting the probability distribution of rockfall intensity is a complex exercise, and should afforded particular attention in the

Fig. 6 Frequency distributions of modeled rockfall bounce heights over the railway track

probabilistic modeling of rockfall trajectories. Furthermore, this suggests that calibrating models using rockfall intensity is an essential task.

Probabilistic modeling in practice

Although it has been stated that it is theoretically rational to treat all rockfall model components probabilistically, there are still some unanswered questions. (1) Will probabilistic modeling add significant value to practical projects such as risk assessment and countermeasure design? (2) Assuming that it is valuable, is it practical (realistic) to carry out probabilistic modeling? (3) Assuming that it is practical, are the costs of implementing probabilistic modeling worth it? We discuss these questions in the following paragraphs, but suggest that the real answers will be found in practice.

The concept of probabilistic modeling proves itself to be valuable, as the precise prediction of rockfall trajectories is not scientifically feasible, although many policy makers wish to see a simple yes-or-no answer. The question then arises as to whether or not adopting a probabilistic approach for all model components in a rockfall model will add quantifiable value in practice. The user-defined parameters will be difficult to constrain and calibrate if the variations in all of the model's components are considered probabilistically. The obvious advantage of representing all of the components in a rockfall trajectory model probabilistically is that every component will display the natural variability or randomness we see in nature. It is therefore important that the practitioner decides which components to adopt a probabilistic approach for, by either comparing modeled results or relying on expertise. Thus, the variations in deterministic parameters are implicitly represented by the variations in probabilistic parameters.

Another issue is whether adopting specific distributions for probabilistic parameters will yield significantly better

results. In order to analyze the effect of probabilistic distributions on probabilistic results, additional numerical experiments were carried out in the same study area (see “[Probability distribution of rockfall intensity](#)”). For the simulations using RocPro3D, we doubled the ranges of the uniform distributions of the parameters used in the previous section. Similarly, for the simulations using Rockfall Analyst, we doubled the angle interval of the launch directions used in the previous section. The spatial distributions of the modeled rockfall bounce heights in the additional experiments are shown in Fig. 7. A close comparison between Figs. 5 and 7 suggests that, in this case, changing the distribution parameters does not significantly change the spatial distributions of rockfall bounce heights. In addition, there is no notable difference in the RocPro3D results when a normal distribution is used instead of a uniform one. These experiments suggest that the difference between the results of different models is more significant than the difference in results obtained with the same model when the probabilistic distributions are varied. Thus, it is the model, not the probabilistic settings, which constrain the probabilistic results. Therefore, understanding the physics of rockfalls and representing it in numerical models should take priority over inspecting the uncertainties of model components. It cannot, however, be confidently stated that finding and applying applicable distributions for probabilistic components will not lead to significantly better probabilistic results and are therefore not necessary. This is because applying different numerical models to different rockfall cases may yield different results which have different implications. Instead, we suggest further analysis of this issue in practice, and that these analyses should be conducted using other rockfall trajectory models and other rockfall case studies.

The last two questions are issues relating to cost–benefit analysis. As has been mentioned, finding applicable distributions for probabilistic parameters will never be an easy task. If it is not realistic to find a probabilistic distribution

Fig. 7 Spatial distributions of rockfall bounce heights modeled by Rockfall Analyst (a) and RocPro3D (b) with probabilistic settings different from those used in Fig. 5 (refer to “[Probabilistic modeling in](#)

[practice](#)” for details). No notable differences can be discerned between Fig. 5 and this figure

that directly characterizes the uncertainty of a parameter, for instance the rock shape, we suggest that the uncertainty in this parameter should be represented by varying other components. It must be emphasized at this stage that, although rockfall model variables incorporate natural uncertainties, they are not fully independent. In fact, some variables—such as the normal restitution coefficient, tangential restitution coefficient, and friction coefficient (within the rebound process)—are not independent of but are correlated with each other. This issue becomes a concern when sampling correlated probabilistic parameters from predefined distributions in trajectory modeling. Therefore, understanding the physics of rockfalls is always fundamental to quantifying the probabilistic characteristics of model variables, as is collecting empirical data from natural or experimental rockfall cases. In practice, of course, time and money are always major concerns. Currently, theoretical and empirical knowledge of the probabilistic characteristics of rockfall model variables is scarce. However, the situation will become better as the relevant theory is developed, technology advances, and the data provided by practice accumulates.

Concluding remarks

The modeling of rockfall trajectories is complicated by not only the inherent stochasticity of rockfalls but also the

epistemic uncertainty associated with numerical modeling. A probabilistic modeling approach is therefore considered necessary, and is adopted in practice by many rockfall trajectory models.

This paper has reviewed the probabilistic modeling of rockfall trajectories in various models. The review has revealed some aspects in which probabilistic modeling has the potential for improvement. It was found that most models do not employ a probabilistic approach for all of their components (i.e., they do not fulfill a systematic probabilistic modeling criterion). In addition, the probabilistic variables in most models are randomly sampled from probability distributions that are not supported by any theoretical or empirical knowledge. The probabilistic modeling of rock shape is especially challenging, primarily because quantifying the effects of rock shape on rockfall dynamics is not a straightforward task. Furthermore, predicting the probability distribution of rockfall intensity is another major challenge that needs particular attention.

A caveat for probabilistic modeling is that the modeling results become hard to constrain if all model components are treated probabilistically. A common question faced by practitioners is when to use a probabilistic approach for a component and when not to. Furthermore, the issues of the value and the cost of the probabilistic modeling of rockfall trajectories remain open. Answering these questions requires putting probabilistic modeling into practice, which in turn helps to develop the theory and advance the

technology associated with the probabilistic modeling of rockfall trajectories.

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