A combined rainfall infiltration model based on Green–Ampt and SCS-curve number

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Abstract:

The Green–Ampt infiltration equation is an incomplete governing equation for rainfall infiltration due to the absence of an inertia term. The estimation of the capillary pressure head at the wetting front is difficult to determine. Thus, a major limitation of the Green–Ampt model is the constant, non-zero surface ponding depth. This paper proposes an integrated rainfall infiltration model based on the Green–Ampt model and the SCS-CN model. It achieves a complete governing equation for rainfall infiltration by momentum balance and the water budget based on the Green–Ampt assumption, and uses the curve number from the SCS-CN method to calculate the initial abstraction, which is used as a basic parameter for the governing equation of the intensity of rainfall loss during the runoff period. The integrated rainfall infiltration model resolves the dilemma for capillary pressure head estimation, overcomes the limitation of constant, non-zero surface ponding depth, and facilitates the calculation of runoff for individual flood simulations. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS Green–Ampt; SCS-CN; runoff; infiltration; soil hydrology

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INTRODUCTION

Calculation of runoff is an important process for individual flood simulations in hydrological investigations. One key problem is rainfall infiltration, which has led to a considerable amount of literature dealing with experimental observations, theoretical analyses, and empirical modelling. R. E. Horton first proposed this concept for storm hydrograph analysis and prediction (Horton, 1941). Since then several infiltration models have been developed, and they can be categorized into three groups: physically based, semi-empirical, and empirical models (Mishra *et al.*, 1999).

The commonly used semi-empirical infiltration models in soil physics and hydrology are Kostikov's model (Kostikov, 1932), Horton's three-parameter model (Horton, 1941), and the SCS-CN (Soil Conservation Service Curve Number) model (USDA-SCS, 1971). Some models were developed for application in homogeneous porous media (e.g. Ghildal and Tripathi, 1987; Hillel, 1998; Jury *et al.* 1991) and others for

ponding conditions (e.g. Parlange *et al.*, 1985; Salvucci and Entekhabi, 1994). Among the various infiltration models in the literature, the Green–Ampt model (Green and Ampt, 1911) is the first physically based equation describing the infiltration of water into the soil. It is widely used, such as in the Soil and Water Assessment Tool (SWAT) and Soil Water-Atmosphere-Plants (SWAP) (Gusev and Nasonova, 1998) land-surface models. However, the parameter h_f (capillary pressure head at the wetting front) in the Green–Ampt model is difficult to estimate, and it has long been believed that this parameter had no obvious physical significance. Although Bouwer (1966) suggested that h_f could be related to measurable soil characteristics, it still imposes difficulties to individual flood simulations.

The main objective of this paper is to provide an integrated rainfall infiltration model based on the Green–Ampt model and SCS-CN model. The Green–Ampt equation is an incomplete governing equation for rainfall infiltration due to the absence of an inertia term. We developed a complete governing equation for rainfall infiltration by momentum balance and the water budget based on the Green–Ampt assumption. The initial abstraction is used as a basic parameter for the governing equation of the intensity of rainfall loss during the runoff period. The initial abstraction was calculated using the SCS-CN method.

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THEORY OF MODELS

The physical content of Darcy's law

Steady water flow in a saturated vertical soil column (Figure 1 A) can be described using Darcy's law:

$$\frac{Q}{A} = \bar{v} = K \frac{h_s + h}{h}. \quad (1)$$

The soil permeability coefficient (K) in Darcy's law is a measure of the rate of water flow through a saturated soil under a given hydraulic gradient in length unit over a given time unit. Under Darcy steady flow, the total pressure is balanced by the total viscous friction force:

$$\rho g A \theta_s \times (h_s + h) = \rho g A \theta_s \times \frac{1}{K} \bar{v} h. \quad (2)$$

The total viscous friction force can be described as:

$$F_v = \rho g A \theta_s \times \frac{1}{K} \bar{v} h. \quad (3)$$

Darcy's law can be derived from the Navier–Stokes equations via homogenization when neglecting the inertia term (Hubbert, 1956). Therefore, the soil permeability coefficient (K) reflects the proportional relationship between the mass flow rate and the total viscous friction force.

Figure 1. Simple geometries representing infiltration of surface water into the soil horizon

Governing equations of rainfall infiltration

The Green–Ampt infiltration model relates the rate of infiltration to measurable soil properties such as porosity, hydraulic conductivity, and moisture content, which is given by:

$$K \frac{h_s + h + h_f}{\theta_s - \theta_0} - \frac{dh}{dt} h = 0. \quad (4)$$

The basic assumption behind the Green–Ampt equation is that water infiltrates into relatively dry soil as a sharp wetting front (Figure 1 B). The Green–Ampt equation (Equation (4)) is an incomplete governing equation for rainfall infiltration due to the absence of an inertia term. The inertia term is more properly understood as shorthand for the principle of inertia as described by Newton in his First Law of Motion. In the absence of inertia, a momentum equation of motion is not strictly conserved. We derive a complete governing equation for rainfall infiltration by incorporating the momentum balance and water budget into the Green–Ampt model.

Based on the total viscous friction force (Equation (3)) and the momentum balance, the momentum equation for rainfall infiltration is given by

$$\rho g A \theta_s \times (h + h_s + h_f) - \rho g A \theta_s \times \frac{1}{K} \bar{v} h = \frac{d(\rho A \theta_s \times h \bar{v})}{dt}. \quad (5)$$

The water budget equation for rainfall infiltration is given by

$$\bar{v} \times dt = (\theta_s - \theta_0) \times dh. \quad (6)$$

By substituting Equation (6) into Equation (5), we obtain

$$\frac{K}{\theta_s - \theta_0} (h + h_s + h_f) - h \frac{dh}{dt} = \frac{K}{g} \frac{d}{dt} \left(\frac{dh}{dt} h \right). \quad (7)$$

To simplify equation Equation (7) we define

$$\theta = \theta_s - \theta_0, \quad (8)$$

to obtain

$$\frac{K}{\theta} (h + h_s + h_f) - h \frac{dh}{dt} = \frac{K}{g} \frac{d}{dt} \left(\frac{dh}{dt} h \right). \quad (9)$$

Equation (9) is the complete governing equation for rainfall infiltration based on the basic assumption of Green–Ampt model. The initial condition for Equation (9) is $h=0$ at $t=0$.

Figure 2. Relative error of the look-up table method for Lambert W function

Using Equation (6) we can compute the soil infiltration rate using

$$f = \bar{v} = (\theta_s - \theta_0) \times \frac{dh}{dt} = \theta \times \frac{dh}{dt}. \quad (10)$$

Runoff occurs when the rainfall intensity (P) exceeds the soil infiltration rate (f) at a point, $f = P$, namely:

$$\frac{P}{\theta} = \frac{dh}{dt}. \quad (11)$$

By substituting Equation (11) into Equation (9), we obtain the design formula for the initial abstraction z_a , which is the cumulative infiltration at $f = P$:

$$z_a = \frac{K(h_s + h_f) - \frac{KP^2}{g\theta}}{P - K}. \quad (12)$$

Using the initial abstraction as a basic parameter, we can rewrite the complete governing equation for rainfall infiltration Equation (9) into a new format. When $f \leq P$, we redefine the cumulative infiltration by:

$$h(t) = z_a + z(t), \text{ and } t = 0, f = P \quad (13)$$

Here, $z(t)$ is the intensity of rainfall loss during the runoff period. By substituting Equation (13) into Equation (9), we obtain

$$\begin{aligned} & \left(\frac{KP^2}{g\theta^2} + z_a \frac{P}{\theta} \right) + \frac{K}{\theta} z - (z_a + z) \frac{d(z_a + z)}{dt} \\ &= \frac{K}{g} \frac{d}{dt} \left(\frac{\theta(z_a + z)}{dt} (z_a + z) \right). \end{aligned} \quad (14)$$

The inertia term is very small relative to the viscous and gravity terms for $f \leq P$. Neglecting the inertia term in Equation (14) gives

$$\left(\frac{KP^2}{g\theta^2} + z_a \frac{P}{\theta} \right) + \frac{K}{\theta} z - (z_a + z) \frac{dz}{dt} = 0. \quad (15)$$

To simplify equation Equation (15) we define the following parameters:

$$A = \frac{KP^2}{g\theta^2} + z_a \frac{P}{\theta}, B = \frac{K}{\theta}, C = z_a. \quad (16)$$

Finally, Equation (15) can be rearranged to obtain:

$$A + Bz - (C + z) \frac{dz}{dt} = 0. \quad (17)$$

Equation (17) is the governing equation for the intensity of rainfall loss during a runoff period.

Calculation of initial abstraction using the SCS-CN model

Assumptions of the Green–Ampt model are: (1) the water content profile is piston-type with a well-defined wetting front, (2) antecedent (i.e. prior to infiltration) water content distribution is uniform and constant, (3) water content drops abruptly to its antecedent value at the wetting front, (4) soil-water pressure head at wetting front is h_f , (5) soil-water pressure head at the surface, h_s , is equal to the depth of the ponding water, and (6) soil in the wetted region has constant properties. These assumptions lead to the following limitations of the Green–Ampt model: (1) lack of homogeneous soil conditions and properties, (2) constant, non-zero surface ponding depth, and (3) in most situations of rainfall or irrigation, the surface is not at constant water content (Ouyang *et al.*, 1998). Also, estimating the value for h_f is difficult as it has long been believed that h_f has no obvious physical basis. Although Bouwer (1966) suggested that h_f can be related to measurable

Table I. Typical mean values for soil hydraulic property parameters (Rawls *et al.*, 1993; Ravi *et al.*, 1998)

	Texture class	θ_s	K_s	CN ^a
		cm ³ /cm ³	m/s	
1	Loamy sand	0.401	1.70 E – 05	50
2	Sandy loam	0.412	7.19 E – 06	55
3	Sandy clay loam	0.330	1.19 E – 05	60
4	Loam	0.434	3.67 E – 06	65
5	Silt loam	0.486	1.89 E – 06	70
6	Clay loam	0.390	6.39 E – 07	75
7	Silty clay loam	0.432	4.17 E – 07	80
8	Sandy clay loam	0.321	3.33 E – 07	85
9	Silty clay loam	0.423	2.50 E – 07	90
10	Clay	0.385	1.67 E – 07	95

^a Not mean values, but typical values used for testing the model in this paper.

Figure 3. Series of curves for infiltration rate during runoff for different soil textures using the integrated rainfall infiltration model

soil characteristics, it is still very difficult to determine and assess whether an h_f value is appropriate.

When the initial abstraction, I_a , is used as a basic parameter to solve Equation (15), estimation of this parameter becomes a key problem. However, this is easily addressed by the SCS-CN method. By using I_a , we resolve the dilemma of determining h_f and overcome the limitations inherent within the Green-Ampt model (i.e. constant, non-zero surface ponding depth and the surface is not at a constant water content). The SCS method can only give an approximate value of the initial abstraction, which needs to calibrate by the actual hydrological sequence data.

The SCS runoff curve number (SCS-CN) method integrates the combined hydrologic effects of soil characteristics, land use, agricultural land management practices, and hydrologic and antecedent soil moisture conditions (McCuen, 1982). The original SCS-CN method was documented in Section 4 of the National Engineering Handbook in 1956 with several revisions through Mishra and Singh (2003). This method is based on a water balance equation and two hypotheses to compute surface runoff in small agricultural watersheds (Mishra and Singh, 1999, 2002, 2003, 2004a, 2004b).

The SCS-CN assumes that I_a is a function of the maximum potential retention (S):

$$I_a = \lambda S \quad (18)$$

I_a is highly variable, but correlated with soil and land cover properties (Patil *et al.*, 2008). In many studies for small agricultural watersheds, an empirical $I_a = 0.2S$ expression is used (SCS, 1985). The S (mm) can be estimated as:

$$S = 25400/CN - 254 \quad (19)$$

where CN is the soil moisture condition I curve number or hydrologic soil-cover complex number ($0 \leq CN \leq 100$). CN

is related to the soil moisture condition II curve number, CN_{II} , with the polynomial:

$$CN = -16.91 + 1.348CN_{II} - 0.01379(CN_{II})^2 + 0.0001177(CN_{II})^3 \quad (20)$$

The CN_{II} is based on the antecedent moisture condition (AMC) II determined by the total rainfall in the 5-day period preceding a storm (USDA-SCS, 1971). Three levels of AMC are used: (1) lower limit of moisture content, (2) average moisture content, and (3) upper limit of moisture content.

Using $I_a = 0.2S$, z_a in Equation (9) can be calculated by:

$$z_a = I_a / (\theta_s - \theta_0) \quad (21)$$

The actual retention, when the initial abstraction is not considered, is:

$$F(t) = (\theta_s - \theta_0)z(t) \quad (22)$$

When $f = P$ for $t = 0$, the direct runoff (Q) yields:

$$Q(t) = Pt - (\theta_s - \theta_0)z(t) \quad (23)$$

Solutions for governing equations

Using Equation (6) we can obtain the soil infiltration rate:

$$f = \theta \times \frac{dz}{dt} = \theta \times \frac{A + Bz}{C + z} \quad (24)$$

The analytic solution of Equation (17) in an implicit form is

$$t = \frac{z}{B} + \frac{BC - A}{B^2} \ln \left(1 + \frac{B}{A} z \right) \quad (25)$$

The analytic solution of Equation (17) in an explicit form is

$$z(t) = -\frac{A}{B} + \frac{BC - A}{B} W \left(\frac{A}{BC - A} \exp \left(\frac{A + B^2 t}{BC - A} \right) \right) \quad (26)$$

Here, $W(x)$ is the Lambert W function which is defined by an inverse exponential function:

Figure 4. Series of curves for cumulative retention during runoff for different soil textures using the integrated rainfall infiltration model

Figure 5. Comparison between the direct runoff simulations from the integrated rainfall infiltration model and SCS-CN model

$$x = W(x)e^{W(x)}. \quad (27)$$

When adopting Equation (26) it is important to calculate the numeric value of the Lambert W function. For the convenience of runoff calculations in hydrological simulation, we need an approximation for $W(x)$ that covers the relevant range. Taking $P = [1.001K, \infty)$ and $t = [0, \infty)$, x can be estimated in the range $[0, -e^{-1})$. Taking $z(t)/z_a = [0.001, \infty)$, $W(x)$ is in the $(-\infty, -1)$ range. The Lambert W function has a branch-point at $x = -e^{-1}$, and in our case $W(x)$ lies at the lower branch. Fritsch *et al.* (1973) proposed an efficient iteration scheme to more accurately compute $W(x)$ based on an initial estimate:

$$W_{n+1} = W_n(1 + e_n). \quad (28)$$

where n is the iteration count and e_n is given by:

$$e_n = \frac{z_n}{1 + W_n} \frac{2(1 + W_n)(1 + W_n + \frac{2z_n}{3}) - z_n}{2(1 + W_n)(1 + W_n + \frac{2z_n}{3}) - 2z_n}, \quad (29)$$

and

$$z_n = \ln \frac{x}{1 + W_n} - W_n. \quad (30)$$

A large number of approximation expressions for the Lambert W function in the Green-Ampt infiltration model have been proposed in the literature (e.g. Fritsch *et al.*, 1973; Brutsaert, 1977; Salvucci and Entekhabi, 1994; Barry *et al.*, 1995; Parlange *et al.*, 2002). For the Green-Ampt model, the relevant range is $-31 < W(x) < -1$ and $-1.067168 \text{ E} - 12 < x < -e^{-1}$. Here, we use the look-up table method to obtain values. Using the numerical method, the step is 0.0001, i.e.

$$W_n = -1 - 0.0001 \times n. \quad (31)$$

Using Equation (26), each x_n is calculated for each W_n . A data table (n , x_n , W_n , x_{n+1} , and W_{n+1}) is stored which

has 30 000 data lines. In practice, for x , using the conditions ($x_n > x \geq x_{n+1}$), we get only one data query result in the table, (m , x_m , W_m , x_{m+1} , and W_{m+1}), then the function $W(x)$ is given by:

$$W(x) = W_{m+1} - \frac{x_{m+1} - x}{x_{m+1} - x_m} (W_{m+1} - W_m). \quad (32)$$

Figure 2 shows that the maximum relative error is less than $1.0 \text{ E} - 8$ for the relevant range of $-1.067168 \text{ E} - 12 < x < -e^{-1}$.

Application of models

The combined rainfall infiltration model based on Green-Ampt and the SCS-CN requires four parameters: θ_s , θ_0 , K_s , and CN. If no measurements are available, θ_s and K_s can be approximated based on soil texture (Table I). The CN value can be obtained from published curve numbers tables (USDA-SCS, 1971).

The Green-Ampt and the SCS-CN models are two classical infiltration models; thus, there is no need to reprove their applicability. In order to demonstrate our combined rainfall infiltration model for different soil types, we set $P = 250 \text{ mm/h}$ and $t = [0, 4] \text{ h}$ to compute a series of infiltration curves (Figure 3) and curves of cumulative retention (Figure 4) during the runoff period, which cover their main ranges.

Although our combined rainfall infiltration model uses the CN (curve number) from the SCS-CN method to calculate the initial abstraction, there are fundamental differences between the two methods. The SCS-CN method is a macroscopic and empirical infiltration model which uses a simple linear relationship between the reciprocal of total actual retention and the reciprocal of total rainfall during the runoff period as:

Table II. Modelling results of artificial rainfall experiments

Code	Location	Rainfall frequency	R ²	
			combined model	Texture class
C1	Qinghai Tianjun	5	0.77	Sandy clay
C2	Qinghai Nachitai	5	0.99	Sandy clay
C3	Qinghai Yanshiping	6	0.86	Sandy clay
C4	Xinjiang Baluntai	11	0.86	Sandy clay
C5	Xinjiang Dahegobi	6	0.83	Sand
C6	Qinghai Golmud	5	0.97	Saline-alkali soil
C7	Qinghai Tianjun	6	0.47	Sandy clay
C8	Xinjiang Dahegobi	5	0.85	Sand
C9	Xinjiang HejinGobi	7	0.72	Sandy clay
C10	Qinghai Yanshiping	6	0.33	Sandy clay
C11	Qinghai Kunlun Mountains	5	0.85	Silty sand
C12	Qinghai Tianjun	8	0.95	Loess
		Average	0.79

Figure 6. Artificial rainfall experiments and modelling by the combined rainfall infiltration model

$$\frac{1}{F} = \frac{1}{S} + \frac{1}{P - I_a} \quad (33)$$

For rainfall infiltration processes, the cumulative infiltration is both the outcome variable and the driving force of gravity; thus, the Lambert W function is closest to the actual infiltration. Figure 5 shows that one runoff curve from the combined rainfall infiltration model crosses the runoff curves from the SCS-CN model at different time points. Overall, the SCS-CN method overestimates the direct runoff.

In order to predict the flood peak in ungauged watersheds, Liu (1978) carried out a series of artificial rainfall experiments in the fields for the period of 1958–1978 using different land surface and soil wetness conditions in many places in China. Six key types of data were recorded for each experiment, including rainfall intensity, initial abstraction duration, initial abstraction, runoff duration, actual retention during runoff period, and steady infiltration rate. We validate the combined rainfall infiltration model by this

artificial rainfall experimental data. Table II and Figure 6 show that the combined rainfall infiltrations model a good accuracy. The chief reason of the errors is that the mean value of the steady infiltration rate is used as the saturated soil hydraulic conductivity for one experimental site, because no measurements are available.

CONCLUSION

We developed an integrated rainfall infiltration model based on the Green–Ampt model and the SCS-CN model. The Green–Ampt equation is an incomplete governing equation for rainfall infiltration due to the absence of an inertia term. We achieve the complete governing equation for rainfall infiltration by momentum balance and the water budget based on the Green–Ampt assumption. The estimation of capillary pressure head at the wetting front, h_f , is difficult to determine. We used the CN (curve number) from the SCS-CN method to calculate the initial abstraction.

The initial abstraction is used as a basic parameter for the governing equation of the intensity of rainfall loss during the runoff period. This resolved the dilemma of determining the h_f value and overcomes major limitations of the Green–Ampt model (i.e. constant, non-zero surface ponding depth and the surface is not at a constant water content). The integrated rainfall infiltration model, which only considers the rainfall infiltration during the runoff period, requires four parameters: θ_s , θ_0 , K_s , and CN . If no measurements are available, θ_s and K_s can be estimated using soil texture, while the CN can be obtained from published values of SCS curve numbers. The integrated rainfall infiltration model significantly facilitates the calculation of runoff for individual flood simulation in hydrological investigations.

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LATIN SYMBOLS

A	$[m^2]$	cross-sectional area of soil column;
CN	$[-]$	soil moisture condition I curve number;
CN_{II}	$[-]$	soil moisture condition I curve number;
$F(t)$	$[m/s]$	cumulative infiltration at the surface;
F_v	$[N]$	total viscous friction force;
$f(t)$	$[m/s]$	infiltration rate in soil;
g	$[m/s^2]$	acceleration of gravity;
h_f	$[m]$	capillary pressure head at the wetting front;
h_s	$[m]$	surface ponding depth;
$h(t)$	$[m/s]$	cumulative infiltration of soil water;
I_a	$[m]$	initial abstraction at the surface;
K_s	$[m/s]$	soil saturated hydraulic conductivity;
P	$[m/s]$	rainfall intensity;
Q	$[m^3/s]$	direct runoff;
S	$[mm]$	potential maximum retention;
t	$[s]$	time;
$W(x)$	$[-]$	Lambert W function;
z_a	$[m]$	initial abstraction depth in soil;
$z(t)$	$[m/s]$	cumulative infiltration depth in soil;
ρ	$[kg/m^3]$	density of water;
λ	$[-]$	coefficient;
θ_s	$[cm^3/cm^3]$	saturated volumetric water content;
θ_0	$[cm^3/cm^3]$	initial volumetric water content.

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