Research on Runoff Sub-model of Non-point Source Pollution Model
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Research on Runoff Sub-model of Non-point Source Pollution Model

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Abstract: In light of the current ability to treat point source pollution, nonpoint source (NPS) pollution has become the primary cause of water pollution. In order to manage and control NPS pollution, we must conduct research on NPS pollution. An effective means for such an endeavor is to construct a mathematical model. However, in the present continuous time and distributed parameter NPS models, such as SWAT, parameter requirements are so numerous that their application is very difficult. In order to make such a model convenient for application, research was first conducted before constructing a new continuous time and distributed parameter NPS pollution model based on hydrodynamics. In this paper, as one of its sub-models, the runoff sub-model is introduced. This sub-model is composed of the SCS model and the water routing model that was constructed by the authors of this paper. This water routing model is based on Saint-Venant equations. Through Laplace transform and inverse Laplace transform mathematical modeling, the outflow hydrograph that is an S-curve was obtained. Then, the authors built the relation between S-curve and water-collecting area coefficient, from which the water routing model is derived. In order to calibrate and validate the new model, the authors applied it in the Guishuihe watershed with satisfactory results. The results show that it has value in application, especially in the area where data are scarce.

Keywords: runoff, nonpoint source pollution, distributed model, hydrodynamics, guishuihe watershed

Introduction

Lack of water resources is a very significant problem in China, with people having to face a serious water resource crisis (Li et al., 1996). Furthermore, water pollution aggravates the crisis. At present, water pollution has become a key factor in limiting national economic development and a serious societal problem, often harming people’s health. With point source pollution now being gradually treated, non-point source (NPS) pollution has become one of the main contributors of water pollution (He et al., 1998). It is, therefore, necessary to conduct research on NPS pollution. An effective means of addressing NPS is to construct a mathematical model.

Up to now, NPS pollution models have gone through two stages. The first is a lumping model, while the second is the stage of a distributed model. A field-scale model for chemicals, runoff and erosion from agricultural management systems (CREAMS) (Knisel, 1980), groundwater-loading effects of agricultural management systems (GLEAMS) (Leonard et al, 1987) and erosion/productivity impact calculator (EPIC) (Williams, 1995) are typical lumping models. Agricultural non-point source pollution model
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(AGNPS) (Young et al, 1989), water erosion prediction project (WEPP) (Flanagan et al., 1995) and soil and water assessment tools (SWAT) (Arnold et al, 1998; Neitsch et al, 2001) are typical NPS pollution models. Lumping models treat the watershed as homogeneous and do not address the spatial distribution of runoff characteristics at different locations in the watershed. Compared with lumping models, a distributed model divides the watershed into individual cells in order to preserve spatial distribution of watershed characteristics and simulated hydrology, sediment and nutrients. Therefore, research on the distributed model has become the focal point of this research on the NPS pollution model. However, present domestic research on the NPS pollution model is still in the stage of a lumping model. Though some distributed models have existed in other countries, there are significant problems when they are applied in China. Some problems come from the model itself. For example, AGNPS is an event-based model that does not reflect temporal variation. Although WEPP and SWAT are continuous time and distributed parameter models, WEPP is only used to simulate the transport of sediment and the SWAT model is still being studied. Its applications are therefore limited. The main reason is that the intensive, site-specific parameter requirements are so numerous in existing continuous time and distributed parameter models that it is very difficult to transfer these models to other watersheds (Heng and Nikolaidis, 1998). If we can reduce the number of parameters to be calibrated in such models, its application will become very easy and simple.

The objective of our research is to construct a new continuous time and distributed parameter NPS pollution model based on hydrodynamics, so as to simulate the transfer and transport of pollutants. This will enable the management and control of NPS pollution. In this paper, the runoff-sub-model, and its application of the new distributed time-variant NPS pollution model, is introduced.

Research on runoff sub-model

SCS (Soil Conservation Service) model (Neitsch et al, 2001)

SCS runoff equation is an empirical model that considers infiltration and transpiration. The model was developed to provide a consistent basis for estimating the amounts of runoff under varying land use and soil types.

The SCS curve number equation is:

\[ Q_{\text{surf}} = \frac{(R_{\text{day}} - I_a)^2}{(R_{\text{day}} - I_a + S)} \]  (1)

where \( Q_{\text{surf}} \) is the accumulated runoff or rainfall excess (mm H\(_2\)O), \( R_{\text{day}} \) is the rainfall depth for the day (mm H\(_2\)O), \( I_a \) is the initial abstractions which includes surface storage, interception and infiltration prior to runoff (mm H\(_2\)O), and \( S \) is the retention parameter (mm H\(_2\)O). The retention parameter varies spatially due to changes in soils, land use, management and slope and temporally due to changes in soil water content. The retention parameter is defined as:

\[ S = 25.4 \left( \frac{1000}{\text{CN}} - 10 \right) \]  (2)

where CN is the curve number for the day. The initial abstractions, \( I_a \), is commonly approximated as 0.2S and equation (1) becomes

\[ Q_{\text{surf}} = \frac{(R_{\text{day}} - 0.25S)^2}{(R_{\text{day}} + 0.85S)} \]  (3)

Runoff will only occur when \( R_{\text{day}} > I_a \).

SCS curve number

The SCS curve number is a function of the soil’s permeability, land use and antecedent soil water conditions. Typical curve numbers for moisture condition II are found in Neitsch et al., 2001. The curve numbers for moisture condition I and III are calculated with the equations:

\[ \text{CN}_I = \frac{4.8 \text{CN}_II}{10 - 0.052 \text{CN}_II} \]

\[ \text{CN}_III = \frac{23 \text{CN}_II}{10 + 0.13 \text{CN}_II} \]
where \( CN_1 \) is the moisture condition I curve number, \( CN_2 \) is the moisture condition II curve number, \( CN_3 \) is the moisture condition III curve number.

**Water routing model**

Saint-Venant equations are basic equations about flood wave motion. In the case of no lateral inflow, the inertia term in dynamic equation is omitted, and for rectangular river cross-sections whose bottom width is \( B \), Saint-Venant equations are reduced to:

\[
B \frac{\partial D}{\partial t} + \frac{\partial Q}{\partial L} = 0 \tag{4}
\]

\[
S_0 - \frac{\partial D}{\partial L} = \frac{Q^2}{K^2} \tag{5}
\]

where \( Q \)—flow rate \((m^3/s)\); \( L \)—distance \((m)\); \( S_0 \)—constant slope, often expressed as bottom slope \( S_L \); \( K \)—modulus of discharge \((m^3/s)\); \( D \)—water depth \((m)\).

Considering that \( K \) is only the function of water depth \( D \), from equation (4) and (5), it can be derived:

\[
\frac{\partial Q}{\partial t} = \frac{K^2}{2QB} \frac{\partial^2 Q}{\partial L^2} - \frac{Q}{KB} \frac{dK}{dD} \frac{\partial Q}{\partial L}
\]

that is

\[
\frac{\partial Q}{\partial t} = \mu \frac{\partial^2 Q}{\partial L^2} - u \frac{\partial Q}{\partial L} \tag{6}
\]

where diffusion coefficient

\[
\mu = \frac{K^2}{2QB} = \frac{Q}{2BS} \]

wave velocity \( u = \frac{Q}{KB} \frac{dK}{dD} \).

If \( \mu \) and \( u \) are constant, equation (6) belongs to constant coefficient differential equation or linear diffusion wave equation. When inflow is instantaneous unit impulse function, outflow can be drawn from the following mathematical equations:

\[
\begin{align*}
\frac{\partial Q}{\partial t} & = \mu \frac{\partial^2 Q}{\partial L^2} - u \frac{\partial Q}{\partial L} \quad (L \geq 0, t \geq 0) \tag{7a} \\
Q(L, 0) & = 0 \quad (L \geq 0) \tag{7b} \\
Q(0, t) & = \delta(t) \quad (t \geq 0) \tag{7c} \\
\lim_{L \to \infty} Q(L, t) & = 0 \quad (t \geq 0) \tag{7d}
\end{align*}
\]

where Eq. (7b) is initial condition, Eq. (7c) and (7d) are upper and down boundary conditions, respectively.

Through Laplace transform and inverse Laplace transform, the expression for instantaneous time travel curve can be derived:

\[
f(t) = \frac{\alpha}{\sqrt{2\pi t}} \exp\left[\frac{-\alpha(t - \beta)^2}{2\beta^2 t}\right] \quad (t > 0) \tag{8}
\]

where \( \alpha = \frac{L^2}{2\mu}, \beta = \frac{L}{u} \).

The outflow hydrograph named as S curve can be derived through Laplace transform: (Rui, 1985; Yangtze River Water Resources Commission, 1993):

\[
S(t) = \frac{1}{2} \left[ 1 - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2} \beta} + \frac{\alpha t}{2 \beta^2}\right) \right] - \frac{\alpha}{2 \beta^2} \left[ 1 - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2} \beta} + \frac{\alpha t}{2 \beta^2}\right) \right] \tag{9}
\]

where \( \operatorname{erf}(\eta) \) or \( G(\eta) \) is Gausse error function. Its expression is:

\[
\operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_{0}^{\eta} e^{-\eta^2} d\eta \tag{10}
\]
Transform S curve into time interval transit curve or concentration factor as follows:

\[ P = S(t) - S(t - \Delta t) \] (11)

which can be also written as:

\[ P_i = S_i - S_{i-1} \quad i = 1, 2, \ldots \] (12)

where \( P_i \) is concentration factor in the time interval number \( i \), \( S_i \) is S curve at the end of time interval number \( i \), \( S_{i-1} \) is S curve at the end of time interval number \( i-1 \).

The outflow hydrograph can be drawn as:

\[ Q_j = \sum_{i=i}^j q_{j-i+1} P_i \] (13)

where \( Q_j \) is concentration flow at the end of time interval number \( j \) (\( m^3/s \)); \( q_{j-i+1} \) is inflow rate in the time interval number \( j-i+1 \) (\( m^3/s \)).

In order to construct a distributed parameter water routing model, the authors studied the relations between parameters in equation (13) and digital elevation model (DEM) cell. From the range of Gause error function: \(-1 \leq \text{erf}(\eta) \leq 1\), the range of \( S_i \) can be obtained from equation (9):

\[ 0 \leq S_i \leq 1 \].

Considering the water-collecting area coefficient of watershed \( A_i \) is total collecting area at the end of time interval number \( i \), \( km^2 \); \( A \) is watershed area, \( km^2 \), we define the S curve at the end of time interval number \( i \) as:

\[ S_i = \frac{A_i}{A} \quad i = 0, 1, \ldots \] (14)

Thus equation (12) can be written in the form:

\[ P_i = \frac{A - A_{i-1}}{A} \] (15)

where \( A - A_{i-1} \) is collecting area in the time interval number \( i \). Now suppose area of unit DEM grid is \( A_{\text{cell}} \) for watershed whose area is \( A \), the number of DEM cells is \( n = \frac{A}{A_{\text{cell}}} \). Therefore, watershed area can be expressed as:

\[ A = n \times A_{\text{cell}} \] (16)

Accordingly,

\[ A_i = n_i \times A_{\text{cell}} \] (17)

in which \( n_i \) is the number of DEM cell contained within collecting area \( A_i \) at the end of time interval number \( i \). Equation (15) can thus be written in the form:

\[ P_i = \frac{A - A_{i-1}}{A} \times (n_i - n_{i-1}) \times A_{\text{cell}} = \frac{n_i - n_{i-1}}{n} \] (18)

In addition, the relation between inflow rate \( q_{j-i+1} \) in equation (13) and surface runoff in unit DEM grid is:

\[ q_{j-i+1} = q_{j-i+1, \text{cell}} \times n \] (19)

in which \( q_{j-i+1, \text{cell}} \) is average runoff in unit DEM grid in the time interval number \( j-i+1 \).

From equation (13), (18) and (19), it can be derived:

\[ Q_j = \sum_{i=0}^j q_{j-i+1, \text{cell}} \times (n_i - n_{i-1}) \] (20)

If \( q_{j-i+1, \text{cell}} \) stands for total runoff in \( (n_i - n_{i-1}) \) DEM cells in time interval number \( j-i+1 \), then \( q_{j-i+1, \text{cell}} \times (n_i - n_{i-1}) \) is equal to \( q_{j-i+1, \text{cell}} \). In this case, the hydrograph at the outlet of watershed can be written as:

\[ Q_j = \sum_{i=0}^j q_{j-i+1} \] (21)

For spatial-variant surface runoff,
\[ q_{j-i+1,k} = \sum_{k=n_{i-1}+1}^{n_i} q_{i-i+1,k} \]  

(22)

where \( q_{j-i+1,k} \) is runoff in certain cell \( k \) among \( n_{i-1} + i \sim n_i \) cells in time interval number \( j-i+1 \).

### Physical meaning of the definition by equation (14)

For spatial-invariant excess rainfall, \( q_{j+i+1} \) in equation (13) has the relation with watershed area \( A \):

\[ q_{j-i+1} = 10^3 l_{j-i+1} A / \Delta t \]  

(23)

where \( j+i+1 \) is depth of excess rainfall in the watershed in the time interval number \( j+i+1 \) (mm), \( \Delta t \) is duration of unit time interval (s), \( 10^3 \) is unit conversion coefficient.

Replace \( q_{j+i+1} \) and \( P_i \) in equation (13) with equation (15) and (23) respectively, then it can be obtained:

\[ Q_j = \frac{10^3}{\Delta t} \sum_{i=1}^{j-i+1} (A_i - A_{i-1}) \]  

(24)

It can be seen that the equation 24 has the same form as the equal flowing time line equation used to calculate watershed transit curve. Therefore, equation (14) has its theoretical basis. The new water routing model, however, is different from the equal flowing time line model in the division of the area where runoff to the outlet has equal flowing time, which is automatic if only the value of parameters \( \alpha \) and \( \beta \) is assigned in the new model and is artificial in the equal flowing time line model.

### Application in Guishui watershed

Researchers (Liang et al., 2001) show that the maximum total phosphorus (TP) concentration in Guanting reservoir area appears in Guishui River, where total nitrogen (TN) concentration is also high. This result indicates that TP and TN pollution in Guishui River is very serious. In addition, the authors analyzed suspended-sediment concentration in Guishui River during July 2002. Before a rainfall event, the concentration was 0.15 kg/m³. During runoff from the rainfall, however, it is up to 1.56 kg/m³. Results show that soil loss of Guishui watershed is very serious. Therefore, we selected Guishui watershed as a research objective.

### Outline of drainage basin

Guishui watershed (controlled by Laojuntang hydrologic station before 1970 and by Yanqingdongdaqiao hydrologic station after 1980) is located in Beijing, with east longitude 115°036’~116°40’ and north latitude 40°14’~ 40°47’. There are seven established rainfall stations in the watershed: Xiangcunying (Laojuntang), Yanqing, Zhangshanying, Kangzhuang, Donghuayuan, Pianpoyu and Yongning. Average elevation is 802.3m, with a minimum elevation of 500m and a maximum elevation of 2833.3m. The main river, the Guishui River, is a branch of the Yongding River drainage.

In the climate sub area, the watershed is located in the temperate warm zone semi-humid region, with blazing hot and rainy summers and cold and dry winters. Normal annual temperature is at 8°C, with a minimum mean temperature of -4°C in January and a maximum mean temperature of 26°C between July and August. Yearly average precipitation is 400-500mm, with 60%~80% of annual precipitation occurring in July, August and September. Evaporation from water surfaces is approximately 1800mm.

Because there are no daily precipitation and sediment data in the recent hydrological yearbook, the authors used 1950s data to calibrate and validate the model. During simulation, soil types and land covers were necessary data for the calculation of runoff, which were obtained from past map collections of the Beijing-Tianjin-Tangshan area. In the maps, the soil types were mostly the same as in the 1950s, as were the retained land covers. Therefore, the maps could be used to obtain soil and land-use types. In addition, the moisture condition of soil was divided according to the precipitation within the antecedent five days.
Hydrologic situation

Figure 1 is a DEM and drainage system map of Guishui River watershed, whose resolution is 100m×100m. According to the digital elevation information of DEM, a three-dimensional view, shown in Fig. 2, was obtained. From Figure 2 it can be seen that the topography of Guishui watershed is very complex. There are mountains, hills and basins.

Before 1970, the hydrometric station of Guishui watershed was located in Laojuntang, making it necessary to draw the watershed controlled by Laojuntang hydrometric station. Based on DEM, the authors obtained the flow direction matrix and flow length by using the hydrological analysis module of ArcView GIS. Then, the watershed controlled by Laojuntang, shown in Figure 3, whose resolution is 1000m×1000m, is generated from flow direction matrix. Drainage area is 850 km². According to flow length away from watershed outlet, the authors put the DEM grid in the order shown in Figure 4, so as to carry out watershed routing.

Calibration and validation

From the hydrological yearbook, it can be seen that heavy rainfall primarily happened in July. Except present months in figures 5, 6, 7 and 8, no heavy rainfall happened in other months. In addition, precipitation and flow rate in 1956 and in 1958 are greater than those in 1957 and in 1960. The flow rate in 1956 is greatest,
therefore the authors selected it to calibrate runoff sub-model. From calibration, it is concluded that optimum values of parameters $\alpha$ and $\beta$ are 42.84 and 165.78 respectively, objective function $F$ is

$$F = \sum \left( \frac{Q_m - Q_{sm}}{Q_m^2} + K(T_m - T_{sm}) \right) = 5.18 \quad (25)$$

where $K$ is a constant which is equal to 0.2, $Q_m$ and $Q_{sm}$ are observed peak flow rate and simulated peak flow rate respectively, $T_m$ and $T_{sm}$ are observed peak time and simulated peak time, respectively. Then, the authors validated this sub-model by use of the flow data in the 1957, 1958 and 1960 hydrologic yearbook. The results are shown in Fig.5~Fig.8.

In order to evaluate the accuracy of main peak flow, it was necessary to make a statistical analysis for main peak flow rate. Its results are given in Table 1, where certainty coefficients are also given. The certainty coefficients were calculated by use of the equation 26:

$$d_Q = \left[ 1 - \frac{\sum(Q_{si} - Q_{oi})^2}{\sum(Q_{oi} - \bar{Q}_{oi})^2} \right] \times 100\% \quad (26)$$
where \(d_q\) is the certainty coefficient, \(Q_{si}\) is the simulated daily flow rate, \(Q_{oi}\) is the observed daily flow rate and \(\bar{Q}_{oi}\) is the average of all daily flow rates.

**Result and discussion**

From hydrologic process modeling, it can be seen that the heavier the rainfall is, the better the simulations are. For example, the rainfall in 1956 and 1958 is very heavy, so the simulated values fit in well with observed data. Whereas rainfall in 1957 is the lightest, simulated values in 1957 are the least accurate.

The results show that this model, calibrated using observed data in 1956 with heavy rainfall and flow rate, is more suitable for the simulation of flow rate in the years with heavy rainfall.

From Table 1, it can be seen that certainty coefficients of the years 1956 and 1958 are quite high, 81.8% and 50.1%, respectively. On the other hand, certainty coefficients for the years 1960 and 1957 are rather low, 25.5% and 5.22%, respectively. In addition, it can be seen from peak errors that the errors of maximum peak flow rate are 13.9%, -14.7%, -4.4% and -31.7%, respectively in 1956, 1957, 1958 and 1960.
Errors of peak flow rate are between –65% and +56%. Therefore, it can be concluded that the accuracy of peak flow rate is very high.

From goodness of fit of flow rate, it can be seen that simulated flow rates fit better with observed flow rates in 1958 than in 1956. But it is the opposite for certainty of simulated value. The reason is that the certainty coefficient is influenced by the peak time error besides goodness of fit. Peak time errors of the year 1958 are greater than those of the year 1956, which results in certainty coefficient of the year 1958 is not too high.

Conclusion

(1) By regarding the watershed as a river net, we put forward a definition of water-collecting area coefficient with specific physical significance. Accordingly, a new distributed water routing model is built based on hydrodynamics.

(2) There are only two parameters α and β that need to be calibrated. The calibrated parameters are so few that it is very easy and simple to apply this runoff sub-model.

(3) Though it is similar to the method of equal flowing time line, the new method divides time zone automatically.

<table>
<thead>
<tr>
<th>Table 1. Statistical results of main peak flow rate</th>
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<td>Data</td>
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<tr>
<td>1956-7-9</td>
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<td>1956-7-17</td>
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<td>1956-7-29</td>
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<td>1960-7-6</td>
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<td>1960-7-16</td>
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Certainty coefficient $d_{0.1956} = 81.8\%$, $d_{0.1958} = 50.1\%$, $d_{0.1960} = 25.5\%$, $d_{0.1957} = 5.22\%$.
(4) Through application in Guishui watershed, good simulated results are achieved. It is shown that this sub-model has value in application, especially in the areas where data are scarce.

About the Authors

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