A discrete square global grid system based on the parallels plane projection

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PLEASE SCROLL DOWN FOR ARTICLE
We developed a direct partitioning method to construct a seamless discrete global grid system (DGGS) with any resolution based on a two-dimensional projected plane and the earth ellipsoid. This DGGS is composed of congruent square grids over the projected plane and irregular ellipsoidal quadrilaterals on the ellipsoidal surface. A new equal area projection named the parallels plane (PP) projection derived from the expansion of the central meridian and parallels has been employed to perform the transformation between the planar squares and the corresponding ellipsoidal grids. The horizontal sides of the grids are parts of the parallel circles and the vertical sides are complex ellipsoidal curves, which can be obtained by the inverse expression of the PP projection. The partition strategies, transformation equations, geometric characteristics and distortions for this DGGS have been discussed. Our analysis proves that the DGGS is area-preserving while length distortions only occur on the vertical sides off the central meridian. Angular and length distortions positively correlate to the increase in latitudes and the spanning of longitudes away from a chosen central meridian. This direct partition only generates a small number of broken grids that can be treated individually.

Keywords: Discrete global grid system; Earth ellipsoid; Parallels plane projection; Projection distortion

1. Introduction

Discrete global grid system (DGGS), in a general term, is a hierarchical structure in which the earth surface is subdivided into a series of discrete grids representing spatial regions or points (Sahr et al. 2003). Map projection-based and polyhedral-based partitions are two broadly employed approaches to generate DGGSs (White et al. 1992). Partitioning on a two-dimensional projected plane according to geographic coordinates is a simple and straightforward way for constructing DGGSs. This method is flexible for choosing a desirable grid size, but its applications are limited due to escalating distortions of area and shape along with latitude increase, particularly in the polar regions (White et al. 1992, Sahr et al.)

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The polyhedral-based partitions have received much more attention in recent years than the map projection-based methods, since they can produce DGGSs with equal-area or near equal-area grids but with less shape distortions. Numerous DGGSs based on the polyhedral partitions have been suggested with different designs and application goals. A detailed summary has been provided by Sahr et al. (2003).

Although various DGGSs obtained by the polyhedral-based partition could exhibit varied characteristics of grid shape, area distortion, cell compactness and spatial indexing (Dutton 1996, Kimerling et al. 1995, White et al. 1998), the essential strategy for constructing grids is common. A DGGS can be designed and developed in five explicit steps (Sahr et al. 2003), including a base polyhedron, a fixed inscribed orientation, a transformation relationship between the polyhedron and the earth surface, a hierarchical partition method and an efficient indexing method for the hierarchical facets (Yuan et al. 2004).

A base polyhedron is usually derived from one of the five platonic solids (Goodchild and Yang 1992, Dutton 1999, White 2000) with equivalent facets composed of congruent convex regular polygons (White et al. 1992). The specification of an inscribed orientation of the polyhedron should consider the symmetry of discrete grids and the relationship between the original faces of the polyhedron and the research areas (White et al. 1992, Randall et al. 2002). Since it is difficult to produce equal-area grids using the direct spherical subdivision of the earth sphere through great circles, map projection-based approaches have been widely applied to create partitions on the original polyhedron facets. Typical projection transformation methods with different distortion features include the gnomonic projection, Lambert azimuthal equal area projection (Yang et al. 2004), Snyder’s polyhedron equal area projection (Snyder 1992), Fuller’s projection (Fuller 1982, Gray 1995) and the zenithal orthotriangular projection (Dutton 1999).

Considering the resolution variations of spatial data sets and the requirement of free scale transformations for spatial modeling and analysis, it is suitable to deploy a hierarchical structure of discrete grids that are produced by the $n$-frequency hierarchy method (Sahr et al. 2003). Several spatial indexing methods have been proposed for efficiently accessing hierarchical grids (Bartholdi and Goldsman 2001, 2004, Yuan et al. 2004).

In this paper, we attempt to develop a new quadrilateral DGGS using direct partitions according to a user-specified grid size on the earth ellipsoid. The discrete grids are structured to be seamless and congruent squares on the two-dimensional projected plane transformed by a new equal area projection called the parallels plane (PP) projection. The justification of this research is examined in the next section. The PP projection is introduced in Section 3. The partition method is discussed in Section 4. The analyses of distortion are presented in Section 5. A potential application is proposed in Section 6. The paper concludes with summative discussions.

2. Justification of the research

There were two motivations underlying this research: (1) we attempt to develop a DGGS that shows evident improvements in comparison with the aforementioned DGGSs, though these DGGSs have demonstrated some salient features and have been employed in actual applications (Randall et al. 2002, Teanby 2006); (2) we want to design a DGGS that supports global, hierarchical and multi-scale data frameworks and related analysis, modeling and visualization in GIS environments.
This new DGGS on the basis of two-dimensional projected squares transformed from the PP projection was designed to break through three limitations shown by most current DGGSs. First, current discrete grids have been largely defined with reference to the earth sphere but not to the earth ellipsoid. The use of the sphere as the earth model is a simplistic approach. The curves comprising of the earth surface in the spherical case are circles. The equation for the sphere in Cartesian coordinates is given as (Earson 1990):

\[ \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = 1 \]  

where \( x \) and \( z \) are two equatorial axes, \( y \) is the polar axis and \( r \) is the radius of the earth circle. However, the (mean) earth ellipsoid is a best approximation of the intermediate projection surface between the irregular earth surface and the mathematically manageable surface of revolution. It is a surface of revolution defined by revolving an ellipse about its minor axis (Earson 1990). The equation for the ellipsoid is given as:

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{a^2} = 1 \]  

where \( a \) is the equatorial radius and \( b \) is the polar radius. The ellipsoid differentiates between the equatorial and polar dimensions, and thus more precisely describes the earth surface compared with the earth sphere. Therefore, the DGGS partitions based on the earth ellipsoid should be more desirable for applications with high data precision requirements (Sahr et al. 2003). The use of the earth ellipsoid as the basic structure to design our new DGGS is the first advantage for supporting more precise data requirements.

There are abundant samples to support this argument. In a GIS-based software development for simulating dynamic urban growth, a hierarchical system of spaces was proposed to include cellular space at city block scale, modeling space at city subdivision or city scale, geographic space at regional scale and environmental (constraining) space at global scale (Xie 1996). Socioeconomic and environmental information are imbedded in spaces of all scales and the disaggregation and aggregation of data across these spaces determine the states and dynamics of urban growth processes (Xie and Batty 2005). Another convincing illustration is the image fusion method that appeared in recent remote sensing research. In the research design for looking into grassland resilience and adaptation to human activities in the Mongolian Plateau, the image fusion of high-resolution Quickbird image data at plant plots, Landsat covering the vicinity of long-term ecological observation stations and MODIS/AVHRR over regional transects and the entire study area is proposed as a key method to measure grassland productivity and its spatial-temporal patterns (Brown et al. 2008). The PP projection-based DGGS with hierarchical partitions of seamless and congruent squares could allow us to infuse datasets taken at different spatial resolutions, such as data from multiple satellite instruments and field validation data. This new type of DGGS could also support a multilevel cell-(grid-)oriented modeling at global scale.

There are other advantages for this ellipsoid-based DGGS, which expects to keep consistent geometric features of facets and represent the earth surface as a seamless and continuous map in the projected plane. Such DGGSs could have potential in GIS applications to summarize and organize the multiple, non-uniformly spaced
measurements over the globe, make comparisons of time-series of globally distributed data (e.g. for detecting land use and land cover, or climate changes) and make statistically meaningful regional comparisons of globally distributed data (Kahn 2008).

3. The PP projection

Spatial analyses and simulations usually require the adoption of a DGGS that should be composed by a series of congruent and regular grids symmetrically tilling the entire objective region. However, it is often difficult or even impossible to create such a DGGS on the ellipsoid directly due to the limitation of geometric features of the earth ellipsoid with inconsistent radii of curvature along the meridian circles. Therefore, we were motivated to combine the ellipsoid with a projected plane as an integrated partition basis so that the arc lengths of parallels and the central meridian remain unchanged during transformation processes. For this purpose, it is necessary to develop a new area-preserving map projection in which no distortion occurs along the parallels and the central meridian when performing the transformation between the earth ellipsoid and this projected plane. The projection we employed in our DGGS is named the PP projection since it is derived from the direct expansion of the parallels and the central meridian.

Let the primary meridian be the central meridian, the equation of transforming the geodetic coordinate of a point defined by longitude \( \lambda \) and latitude \( \phi \) to the Cartesian projected coordinate \((x, y)\) can be written as:

\[
\begin{align*}
  x &= \lambda \frac{a \cos \phi}{(1 - e^2 \sin^2 \phi)^{1/2}} \\
  y &= \int_0^\phi \frac{a(1-e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}} d\phi
\end{align*}
\]

where \( a \) is the semi-major axis and \( e \) is the first eccentricity of the earth ellipsoid.

Detailed discussions of the PP projection have been provided in our previous paper and thesis (Zhou et al. 2006, Ma 2006). A brief summary of the PP projection characteristics are given as follows: (1) PP is an area-preserving projection; (2) parallel circles are projected as parallel straight segments without length distortions; (3) meridian circles are projected as a series of sine or cosine curves, and the length distortion has an increasing tendency with the distance further away from the equator and the central meridian; (4) the angular distortion is getting higher with increasing longitude and latitude.

4. The partition method

The method of direct partition has been employed in this study to create two dimensional congruent discrete grids, of which the shapes are expected to be uniform squares. On the projected plane, a square grid \( A'B'C'D' \) with size \( d^2 \) (in \( \text{km}^2 \) and \( d \) is the grid dimension) can be easily obtained by placing linear parallels with the same interval (Figure 1). The determination of its corresponding quadrilateral facet \( ABCD \) on the earth ellipsoid, however, needs more sophisticated inverse computation (Zhou et al. 2006, Ma 2006). Since the parallels are projected as the straight segments and have no length distortion during the transformation through the PP projection, the arc length of \( AB \) and \( CD \) derived from the parallels is...
equal to the segment length of $A'B'$ and $C'D'$. In order to keep the projected shape of the quadrilateral on the earth ellipsoid as a perfect projected square, it should satisfy a primary condition that the arc length of any segment of parallels formed by sides $AB$ and $CD$ equals $d$ strictly (such as $A_nB_n$ in Figure 2). It means that four vertices of $ABCD$ are easy to find through the partition defined by the fixed arc length of parallels circles. Therefore, the partition process is also straightforward on the earth ellipsoid through finding two pairs of ellipsoidal points on the same parallel circle, such as $A-B$ and $C-D$ (Figure 2). The arc length of parallels between a pair of points should be equal to the side dimension of the grid, $d$. Using this partition method, we can obtain a DGGS with any resolution and a given central meridian (Figure 3).

Although it is easy to perform a transformation between the projected and the ellipsoidal grids for the points located on the corner of the horizontal side (such as $AB$ or $CD$ in Figure 2), it is more difficult to formalize computational equations for the points on the vertical sides of ellipsoidal grids (e.g. $AD$ and $BC$ in Figure 2), because the vertical side is neither a regular spherical nor an ellipsoidal curve and its length is not a constant. We have developed an approximation method for computing the coordinate transformation and curve length for these vertical sides. Let $(x_1,y_1)$ and $(x_2,y_2)$ be the projected Cartesian coordinates for point $B'$ and $C'$ (Figure 2), respectively. It is obvious that $x_1=x_2$ and all points on the side of $B'C'$ have the same $x$-coordinate. Suppose that grid $A'B'C'D'$ is labeled at the $i$th row and the $j$th column. The quantitative relationship between the projected coordinates and the side dimension of the grid can be written as:

$$
\begin{cases}
    x_1 = id, y_1 = (j-1)d \quad \text{(for $B'$)} \\
    x_2 = id, y_2 = jd \quad \text{(for $C'$)}
\end{cases}
$$

Figure 1. Schematic diagram of partition for discrete global grid system on the surface of Earth ellipsoid (a) and on the two-dimensional projected plane (b) transformed by the parallels plane (PP) projection.
For any given point \( P(x, y) \) on the side of \( B'C' \), its corresponding point \( P(\lambda, \phi) \) on the ellipsoidal arc \( BC \) should satisfy the projection transformation equation (3). In order to avoid the sophisticated inverse computation, we have used the following equation to replace the latitude \( \phi \) in equation (3) with the geocentric latitude \( \omega \) (in
Thus, equation (3) can be altered as follows:

$$x = \lambda \rho \cos \omega$$  

(6)

where $\rho$ is the vector radius connecting the point $P$ and the center of the earth ellipsoid and can be expressed as a function of geocentric latitude:

$$\rho = a \sqrt{\frac{1-e^2}{1-e^2 \cos^2 \omega}}$$  

(7)

As shown in Figure 4, $\rho$ can also be approximated as an average of vector radii $\rho_1$ and $\rho_2$ for point $B$ and $C$ respectively if arc $BC$ is treated as a spherical curve with a radius of the curvature $\rho$:

$$\rho = \frac{1}{2} (\rho_1 + \rho_2) = \frac{1}{2} \left[ \frac{a \sqrt{1-e^2}}{\sqrt{1-e^2 \cos^2 \omega(\varphi_1)}} + \frac{a \sqrt{1-e^2}}{\sqrt{1-e^2 \cos^2 \omega(\varphi_2)}} \right]$$  

(8)

Therefore, the geocentric coordinate of $P$ denoted by $(X,Y,Z)$ can be obtained by combining equations (6) and (8) as:

$$\begin{cases}
X = \rho \cos \omega \cos \lambda = x \cos \lambda / \lambda \\
Y = \rho \cos \omega \sin \lambda = x \sin \lambda / \lambda \\
Z = \rho \sin \omega = (x^2 + y^2)^{1/2} / \lambda
\end{cases}$$  

(9)

This equation is the analytic expression of the vertical side of grids on the earth ellipsoid, and also enlightens the mathematical relationship of the transformation between the projected squares and the ellipsoidal grids. This equation implies that a vertical arc of ellipsoidal grids, such as $AD$ or $BC$ except on the central meridian, is a sophisticated spatial curve. The actual length of the vertical arc can be computed through the spatial curve length equation:

Figure 4. Definition of average vector radius for vertical side of the ellipsoidal facet.
Substituting equation (9) for $X, Y$ and $Z$ in equation (10) yields:

$$|BC| = \int_{\hat{\lambda}_i}^{\hat{\lambda}_f} \sqrt{\left(\frac{\partial X}{\partial \hat{\lambda}}\right)^2 + \left(\frac{\partial Y}{\partial \hat{\lambda}}\right)^2 + \left(\frac{\partial Z}{\partial \hat{\lambda}}\right)^2} \, d\hat{\lambda}$$

(10)

Let $L(\hat{\lambda})$ denote:

$$L(\hat{\lambda}) = \int_{\hat{\lambda}_i}^{\hat{\lambda}_f} \sqrt{1 + \left(\frac{k}{\lambda^2 - (x/\rho)^2}\right)} \, d\hat{\lambda} \quad \text{(where } k = \frac{x}{\rho})$$

(12)

Then, equation (11) can be rewritten as the conditional expressions as: when $k^2 \geq 1$,

$$L(\hat{\lambda}) = \frac{x}{2} \ln \left[ \frac{\sqrt{\hat{\lambda}^2 - k^2 + 1} + \sqrt{\hat{\lambda}^2 - k^2} - k \sqrt{\hat{\lambda}^2 - k^2 + 1}}{\sqrt{\hat{\lambda}^2 - k^2 + 1} - \sqrt{\hat{\lambda}^2 - k^2}} \right] - \frac{x\sqrt{k^2 - 1}}{2k} \ln \left[ \frac{\sqrt{(\hat{\lambda}^2 - k^2)(k^2 - 1)} + k \sqrt{\hat{\lambda}^2 - k^2 + 1}}{\sqrt{(\hat{\lambda}^2 - k^2)(k^2 - 1)} - k \sqrt{\hat{\lambda}^2 - k^2 + 1}} \right]$$

(13)

when $0 \leq k^2 < 1$,

$$L(\hat{\lambda}) = \frac{x}{2} \ln \left[ \frac{\sqrt{\hat{\lambda}^2 - k^2 + 1} + \sqrt{\hat{\lambda}^2 - k^2}}{\sqrt{\hat{\lambda}^2 - k^2 + 1} - \sqrt{\hat{\lambda}^2 - k^2}} \right] - \frac{x\sqrt{1 - k^2}}{k} \tan^{-1} \left( \frac{k \sqrt{\hat{\lambda}^2 - k^2 + 1}}{\sqrt{(\hat{\lambda}^2 - k^2)(k^2 - 1)}} \right)$$

(14)

Thus, the arc length of $BC$ can be simplified as:

$$l_2 = |BC| = L(\hat{\lambda}_2) - L(\hat{\lambda}_1)$$

(15)

Especially, the curve length of $BC$ equals the length of the meridian curve truncated by points $B$ and $C$ while the vertices are located on the central meridian. It suggests that no length errors occur along the central meridian during the processes of transformation and partition.

The direct partition method for constructing square grid systems on the projected plane has induced two special kinds of broken grids as their shapes differ from the regular grids discussed above. One type of broken grids is caused by the parallels converging at the north and south poles. As shown in Figure 5, sides $K'N'$ and $J'N'$ of broken grid $K'N'J'$ are the partial segments of the planar meridian projected as a sinusoidal curve with the maximum longitude difference from the central meridian. The corresponding area $K'N'J'$ is the surface area of the ellipsoidal cap. The occurrence of the other type of broken grids is due to the fact that the arc lengths of parallels cannot be exactly divided by the side length of the chosen grid. Let us look at grid $A'B'EF'E'$ in Figure 5. Side $A'B'$ is a regular vertical side as aforementioned, but the other vertical side $E'F'$ is a part of the sinusoidal curve projected from the meridians. Its projected length $l$ (in kilometer) can be computed though the equation:
Thus, the side length of $E'F'$ is:

$$l(\lambda, \varphi) = a(1-e^2) \int_0^\varphi \frac{(1+\lambda^2 \sin^2 \varphi)}{(1-e^2 \sin^2 \varphi)^{3/2}} d\varphi$$

(16)

Thus, the side length of $E'F'$ is:

$$|E'F'| = l(\lambda_E, \varphi_E) - l(\lambda_F, \varphi_F)$$

(17)

where $(\lambda_E, \varphi_E)$ and $(\lambda_F, \varphi_F)$ are the geodetic coordinates of the corresponding ellipsoidal points for $E'$ and $F'$, respectively. Comparing with the normal horizontal sides, sides $A'E'$ and $B'F'$ will display length variations, which can be computed by the mod function:

$$|A'E'| = |x_1| \mod(d)$$

$$|B'F'| = |x_2| \mod(d) + d$$

(18)

5. The analyses of distortions

5.1 Length distortion

According to the partition method discussed above, planar facet $A'B'C'D'$ is a square with size of $d^2$ but its corresponding ellipsoidal grid $ABCD$ is an irregular quadrilateral enclosure. There are no length distortions on the parallels during the PP projection transformation. The horizontal sides on both the projected plane and the earth ellipsoid can be generated by the direct equidistance partition without length distortions. However, the curve lengths of the vertical sides of the ellipsoidal grids except on the central meridian are varying. Their changes along the longitude and latitude can be described through length ratio $\mu$, which is defined as:

$$\mu = \frac{d}{l_2}$$

(19)

The actual curve length of vertical side $l_2$ relates to longitude $\lambda$, latitude $\varphi$, grid side dimension $d$ and the projected x-coordinate $x_1$ of vertex B. Hence, equation (15)
can be rewritten as a function of these parameters as:

\[ l_2 = |BC| = F(\lambda, \varphi, d, x_1) \]  \hspace{1cm} (20)

This equation indicates that the length ratio is determined by four different variables. The possible impacts of these variables on the length ratio can be explored under three scenarios:

1. While \( d \) and \( x_1 \) are given constants, the vector radius (the geocentric latitude) can be expressed as a function of longitude \( \lambda \) according to equations (6) and (7):

\[
\rho = \rho(\omega) = \rho(\lambda) \\
\omega = \arccos \left[ \frac{x_1}{\sqrt{x_1^2 e^2 + \lambda^2 d^2 (1 - e^2)}} \right] \hspace{1cm} (21)
\]

Associating equations (20) and (21), we can obtain:

\[
\frac{\partial F(\lambda, \varphi, d, x_1)}{\partial \lambda} = \frac{x_1}{\lambda} \sqrt{1 + \frac{\rho^2}{\lambda^2 \rho^2 - x_1^2}} > 0 \hspace{1cm} (22)
\]

This is a monotone function for \( \lambda \), which implies that the length ratio is positively correlated to longitude on fixed parallels and \( x \)-coordinate with chosen grid dimension (Figure 6).

2. While \( d \) and \( x_1 \) are given constants, using equation (6) to replace \( \lambda \) in equations (11) and (20) yields:

Figure 6. Length distortion of vertical sides against longitude (in degree) on five different parallel circles (referred to Krasovsky ellipsoid and \( d=1 \) km).
The effect of latitude on the length ratio can be derived from a differencing operation of this equation:

\[
F(\lambda, \varphi, d, x_1) = \int_{\phi_1}^{\phi_2} \sqrt{x_1^2 \tan^2 \varphi + \rho^2} d\varphi
\]

Equation (24) implies that the length ratio increases with latitude (Figure 7).

(3) Considering effect of the side dimension of grid \(d\) on the length ratio, let \(g(\varphi)\) denote

\[
g(\varphi) = \sqrt{x_1^2 \tan^2 \varphi + \rho^2(\varphi)}
\]

where \(\rho\) is expressed as the function of \(\varphi\). According to the mean value theorem for integration, there exists \(\phi_0 \in [\phi_1, \phi_2]\). Hence, equation (23) can be rewritten as:

\[
F(\lambda, \varphi, d, x_1) = \int_{\phi_1}^{\phi_2} g(\varphi)d\varphi = g(\phi_0)\Delta \varphi
\]

According to the properties of meridian and the PP projection, a higher chosen value of \(d\) can cause an increase of \(\Delta \varphi\) (e.g. the latitude difference of two vertices). Thus, equation (26) shows that the change ratio of curve length of vertical sides is a positive correlation function of the grid dimension (Figure 8).

![Figure 7. Length distortion of vertical sides against latitude (in degree) with three different x-coordinates (referred to Krasovsky ellipsoid and \(d=1\) km).](image-url)
5.2 Angular distortion

The intersecting angle between the vertical and the horizontal sides is \( \pi/2 \) exactly on the projected plane (Figure 2). However, its corresponding angle on the earth ellipsoid is more complicated. Let \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \) be the intersecting angles formed by the tangent lines of curve \( AB \) and \( AD \) at point \( A \), as shown in Figure 2. It is obvious that \( \theta_1 = \theta_3 \) and \( \theta_2 = \theta_4 \). The definition of \( \theta_1 \) can be expressed as

\[
\tan \theta_1 = \lim_{|AS| \to 0} \frac{|KS|}{|AS|} = \lim_{|d\lambda| \to 0} \frac{Md\varphi}{rd\lambda} = \frac{M d\varphi}{rd\lambda} \bigg|_A \tag{27}
\]

where \( M \) is the radius of curvature in the meridian and \( r \) is the radius of parallels. In this equation, the value of \( d\phi/d\lambda \) can be derived from the expression of the radius of curvature in the prime vertical \( N \):

\[
\frac{d\varphi}{d\lambda} = \cos \varphi \frac{1 - e^2 \sin \varphi}{\lambda(1 - e^2) \sin \varphi} \tag{28}
\]

Substituting this equation into equation (27) leads to:

\[
\tan \theta_1 = \left. \frac{M d\varphi}{rd\lambda} \right|_A = \frac{1}{\lambda \sin \varphi} \bigg|_A \tag{29}
\]

Let \( \varepsilon_1 = \pi/2 - \theta_1 \) denote the angular distortion of \( \theta_1 \), we can obtain:

\[
\tan \varepsilon_1 = \frac{\lambda}{\sin \varphi} \bigg|_A \tag{30}
\]

The relationship between the angular distortion \( \varepsilon_2 \) for \( \theta_2 \) and \( \varepsilon_1 \) can be written as:

\[
\varepsilon_2 = \frac{\pi}{2} - \theta_2 = -\varepsilon_1 \tag{31}
\]

Thus, we can use the absolute value of \( \varepsilon_1 \) to represent the angular distortion of \( \varepsilon_2 \) (Figure 9). This analysis indicates that the angular distortion escalates with increasing latitude and longitude but does not depend on the grid dimension.
6. A potential application: a data model supporting agent-based urban modeling

The proposed DGGS could act as a preferred data structure to support the agent-based simulation of Desakota, an emergent urban form, in the Suzhou-Wuxian Region in China (Xie et al. 2007). Three distinct types of agents were deployed in this simulation: (1) developer agents such as individual entrepreneurs, small corporations, town- and village-owned enterprises, and privately-operated businesses at the micro-level; (2) township agents that combine town–village developers, municipality developers, external developers (including investors from abroad and other China municipalities) at the meso-level; (3) regional agents encompassing municipal, provincial, national policy makers at the macro-level. The individual developer agents drive urban development process at the cellular level. The township agents determine accessibilities, and land costs reflected through land suitability. The regional agents control growth management policies that affect the location advantages or disadvantages (the township competition index) directly and the behaviors of the developer and the township agents indirectly.

From the perspective of simulation in space, the developer agents begin by considering development in the cellular neighborhood of each master agent activating a process we call neighborhood allocation. In other words, a developer agent begins by considering cells in the immediate band of eight cells around the master agent, in the Moore neighborhood, and if no suitable cell is found, then the agent considers the next band of cells, and so on until a suitable cell is found (Xie et al. 2007). The township agents examine the land suitability and the land cost in terms of the existing land use and the distance from a town center respectively, the accessibility to economic centers and the accessibility to transportation facilities. The regional agents define a policy index, which is in effect ‘a township competition index’.

From the terminology of the DGGS (Figure 10), the agent grids (Figure 10e) represent individual developer agents and could be used to store attributes that

Figure 9. Angular distortion of grids against longitude (in degree) on five different parallel circles (referred to Krasovsky ellipsoid).
determine the behaviors of the developer agents. The neighborhood grids (Figure 10d) will define the spatial extents of the activities of individual developer agents. In other words, the developer agents are usually searching within these grids to make judgment about the land suitability and the land cost. The descriptive data about the land and the information about the agents’ search rules could be better stored in the neighborhood grids. Moreover, the township/city divisional grids are adequate to describe the accessibilities to the economic centers and the transportation facilities. The meso-scale influences and underlying variables could be managed along with the township/city divisional grids. Finally, the regional and global grids determine the competition index that also reflects the policy impacts. Therefore the data explaining the governmental policies, the regional economic competition and cooperation, and the external investment and influence could be better handled over these grids. In brief, the DGGS proposed in this paper could serve as an alternate data structure and modeling framework to support this agent-based simulation of Desakota in Suzhou, China.

Figure 10. An illustration of hierarchical square DGGSs with the central meridian 115°E on the projected plane transformed by PP projection: (a) the global grids covering China with $d=100$ km, (b) the regional grids covering Jiangsu Province with $d=10$ km (fine grid) and $d=100$ km (coarse grid), (c) the township/city-divisional grids covering Suzhou City with $d=3$ km (fine grid) and $d=10$ km (coarse grid), (d) the neighborhood grids covering the urban area of Suzhou with $d=1$ km (fine grid) and $d=3$ km (coarse grid), (e) the agent grids representing individual agents with $d=100$ m (TM remote sensed image with spatial resolution of 30 m).
7. Conclusions

In this study, we have explored the construction of a congruent square discrete grid system on the basis of a specially projected plane. Unlike most other approaches that start with one of the regular polyhedrons or the truncated polyhedrons for dividing the earth sphere, we have employed a direct partition method to carve up the surface of the earth ellipsoid. Thus, our method derived from the earth ellipsoid more accurately portrays the earth surface compared with other methods based on the earth sphere. Second, the surface of the earth ellipsoid is projected into a two-dimensional discrete square grid system in order to produce a seamless DGGS. The partitioning of the square grids is straightforward by placing parallels in equal distances. The locations of four vertices of the quadrilateral facet on the earth ellipsoid are also directly obtained using a fixed arc length along the parallel circles since this arc length remains unchanged during the process of projection transformation. A notable advantage of this direct partition approach is that the grid resolution can be chosen freely according to specific application demands. By comparison, the common limitation of maintaining the $n$-fold relationship between adjacent level grids that is required by the recursive hierarchical partition methods is avoided by our method.

Third, this newly developed DGGS generates less error propagation for spatial data. The parallels plane (PP) projection derived from the expansions of the central meridian and the equator provides an area-preserving transformation between the planar grids and the corresponding ellipsoidal grids. As part of the parallels, both horizontal sides of this newly constructed DGGS grid have the same length and no length distortions happen during transformation. The distortions of the two vertical sides and their intersecting angles except the central meridian, however, are getting worse with increasing latitude and the spanning of longitude away from the central meridian. The severities of angular and vertical side distortions could be reduced through carefully choosing the central meridian. Furthermore, this DGGS is causing some broken grids near two polar regions and the converging areas. However, the negative impact of broken grids should be trivial because they are few in numbers. Finally, this congruent square grid system characterized with recursive hierarchical partition and at any resolution provides a preferred data structure and modeling framework to support cell-based models and simulations. This DGGS should be a powerful alternate data structure for integrating multiple-scale and non-uniformly spaced observations over the large study area or the globe to make temporal and spatial comparative studies.

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